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AMRL-TDR-62-140 (I)

RESEARCH ON SOUND PROPAGATION IN SOUND-ABSORBENT DUCTS WITH SUPERIMPOSED AIR STREAMS

VOLUME I

TECHNICAL DOCUMENTARY REPORT NO. AMRL-TDR-62-140 (I)

December 1962

Biomedical Laboratory
6570th Aerospace Medical Research Laboratories
Aerospace Medical Division
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

Contract Monitor: William C. Elrod, Captain, USAF
Project No. 7231, Task No. 723104

[Prepared under Contract No. AF 61(052)-112
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2. Sound Transmission (sound and acoustics)
3. Acoustics (sound and acoustics)

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FOREWORD

This report is Volume I of a series of reports on sound propagation in sound-absorbent ducts with a superimposed air stream prepared by Göttingen University, Göttingen, Germany, for the Bioacoustics Branch, 6570th Aerospace Medical Research Laboratories, Aerospace Medical Division, under Contract AF 61(052)112. The work was performed under Project 7231, "Biomechanics of Aerospace Operations," Task 723104, "Biodynamic Environments of Aerospace Flight Operations." Principal Investigators for Göttingen University were Dr. Erwin Meyer and Dr. Fridolin Mechel. Technical and administrative personnel monitoring this effort have included Dr. H. von Gierke, Capt. W. Elrod, R. G. Powell, and J. N. Cole.

Initial research on this investigation commenced in June 1958.

ABSTRACT

This report, Volume I, discusses the possible effects of flow on airborne sound attenuation in ducts. The theoretical part results in a simple attenuation formula which considers the following: change of the wavelength due to convection of the sound field, change of the sound pressure at the wall caused by the flow profile, change of the characteristic absorber properties by nonlinear effects, and sound scattering by vortices. With porous absorbers another effect is caused by the different curvature of the phase plane at the boundary of the absorber. Measurements with a porous absorber and with damped Helmholtz resonators show reduction of the attenuation for sound propagation in the direction of the flow and an increase of the attenuation for sound propagation against the flow. With the help of pseudo-sound in flow and of partial waves in ducts with a periodic boundary structure, the sound amplification found in ducts coated with reactive absorbers can be explained by analogy to traveling wave tube amplification phenomena. This analogy was confirmed by measurements on resonant absorbers.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

Jos M Quashnock

JOS. M. QUASHNOCK
Colonel, USAF, MC
Chief, Biomedical Laboratory

TABLE OF CONTENTS

List of Symbols.

p	pressure
v	particle velocity
I	sound intensity
N	sound power
J	mass flow density
$\vec{n}$	energy flow density
$\bar{v}_n$	particle velocity normal to wall
$\bar{L}$	wall admittance
V, V_o	flow velocity
u	phase velocity
c	adiabatic velocity of sound
$M = V/c$	Mach number
$\mu = (1 - M^2)^{-1/2}$	
ω	radiant frequency
f	frequency
λ	wavelength
$k = \omega/c$	wave number
$\beta = \omega/u$	phase constant
α	attenuation per unit length
α'	attenuation per wavelength
f_c	resonance frequency
m	vibrating mass of a resonator
ρ	density of air
h	effective height of duct
L	length of period of duct wall
n	partial wave index
A_n	amplitude of n-th partial wave
σ	porosity of absorber
r	flow resistance of absorber; vortex radius
$\eta = r/\omega\rho$	
ϵ	ratio of air densities inside and outside of porous material

ϑ	angle between wave front and absorber surface
A	constant
F	field quantity
$j = \sqrt{-1}$	
z	coordinate in flow direction in stationary coordinate system
x, y	coordinates normal to z
x', y', z', t'	transformed coordinate system
∇	nabla operator
Δ	Laplacian operator
Δ_{tr}	Laplacian operator in the coordinates transverse to z-axis

Summary

Possible effects of flow on air-borne sound attenuation in ducts are discussed with the help of a simple attenuation formula. These effects are: Change of the wavelength due to convection of the sound field, change of the sound pressure at the wall caused by the flow profile, change of the characteristic absorber properties by non-linear effects and sound scattering by vortices. With porous absorbers another effect is caused by the different curvature of the phase plane at the boundary of the absorber. Measurements with a porous absorber and with damped Helmholtz resonators show reduction of the attenuation for sound propagation in the direction of the flow and an increase of the attenuation for sound propagation against the flow.

With the help of pseudo-sound in flow and of partial waves in ducts with a periodic structure of the boundary the sound amplification found in ducts coated with reactive absorbers can be explained in analogy to the travelling wave tube amplification. This analogy was carried out and was confirmed by measurements on resonance absorbers.

Part I Experimental Set-up

A. Introduction

Attenuation of air-borne sound in ducts with superimposed air flow is a prerequisite in many technical applications. For example in ventilation ducts in buildings the noise generated by the blower should not enter the ventilated rooms. The ducts should also not allow noise from the outside to penetrate into the building. These ducts should therefore attenuate the air-borne sound propagating against the flow direction. In test rooms for jet engines the very high noise level of the jet stream has to be reduced before the latter leaves the test room. In this case the ducts should absorb the air-borne sound propagating in the direction of the flow.

Since no results of systematic investigations of the sound attenuation in flow ducts were available it was usually attempted to determine proper attenuation values according to the many accurate technical, experimental and theoretical experiences for sound attenuation in ducts with resting air. Flow was only taken into consideration with respect to the technical requirements of strength and heat stability of the absorbing material and structures. Sometimes heuristic considerations of the surface structure of the absorbers and of the lay-out of the ducts were applied.

In some cases it was found that the attenuation in such absorber arrangements was different for the two directions of sound propagation and - which was especially disappointing - that the effect of the absorber was much lower than expected.

During their course of development, acoustics and aerodynamics, both investigating movements of air, have become different scientific fields with characteristic laws of development, which for a long time were rather divergent. In our time the application of high flow velocities combined with high sound levels leads to a closer cooperation in solving many common problems. As will be shown in the work at hand the investigation of air-borne sound attenuation in flow ducts leads to the formulation of questions of gasdynamics. In this work the problem was to be treated under two guiding principles: In the first place an experimental basis was to be secured for an eventual theoretical treatment of the experienced phenomena; the complexity of these phenomena can only be open for a theoretical discussion at tolerable expenditure if experimental results lead a way to a logical solution. Furthermore it was to be tried how far the analogy - obvious for an acustician - with electromagnetic propagation mechanisms would lead in the interpretation of the experimental results. Therfore the analogy with the propagation of electromagnetic waves along an electron beam in the travelling wave tube was kept in mind during the investigation of the sound propagation in ducts with streaming air and especially in connection with the experienced sound amplification.

In an earlier work [1, 2] attenuation measurements were made in ducts with absorbing walls and with a superimposed air flow. It was found that the effect of the flow on the propagation of sound waves in such ducts can be summarized into three observations:

1) The change of the attenuation of porous absorbers and highly damped resonance absorbers,
2) marked attenuation minima or signal amplification resp. with undamped resonators,
3) self-excitation of ducts with undamped resonators.

With respect to the first two points this work is the continuation of the work quoted above; the results of this earlier work will be used here several times. The author does not know of any other experimental investigation concerning the attenuation of air-borne sound in flow ducts. The applicability of the theoretical treatment of this problem by Pridmore - Brown [3] will be tested with the experimental results contained in this work.

B. Measuring Technique

1) Wind tunnel

The arrangement of the wind tunnel is essentially the same as described in [1, 2]. Fig.1 gives a schematic view of the set-up. A single-stage radial blower with a motor of 7.5 kW electrical power input generates the air flow. The flow velocity was adjusted with the help of a throttle valve at the air intake. The pressure tube was insulated from structure-borne sound from the blower by rubber collars. Such insulators were used also at different other places in the duct to prevent disturbances in the test tube by structure-borne sound generated by the flow in the ducts. The air from the blower is penetrating a first silencer of 1.5 m length (not entered into Fig.1) which is constructed in a way to suppress the more pronounced frequency components of the blower noise. In this silencer the air was led through a tube of perforated sheet metal through a transversely coffered box. The single compartments were tuned to the noise components to be attenuated by proper filling with rock wool of different density. After this silencer almost only the white noise of the flow remains. Blower and first silencer were situated in a side room with a brick partition wall. In the measuring room the flow penetrated another silencer of 1.5 m length (see Fig.1) which consisted of an arrangement of parallel plates of rock wool. After this silencer the free cross-section of the duct was continually reduced to the entrance of the test tube. In front of the test tube the generators for the acoustical signal and a nozzle for velocity measurements were located. For measurements of the sound propagation against the flow direction the sound sources lay at the other end of the test tube. The flow velocity was partly measured with a Pitot tube penetrating about 70 cm into the test duct from the far end. The test duct had a length of 2.4 m and a flat rectangular cross-section. Typical values of the cross-section are 33 mm height and 99 mm width. The absorber under test was put on the upper broad side of the duct. At the end of the duct a diffusor of 1.5 m length made out of rock wool reduced the reflection at the duct termination, the cross talk into the duct and the noise from the exhaust.

For measurements of attenuation and phase velocity of the sound wave in the duct the probe tube (8 mm diameter) of a moving coil microphone acoustically matched to the probe tube was drawn through the duct by a synchronous motor. For minimal noise level a probe tube with lateral openings smoothly covered with copper gauze proved to be useful. The distance of the holes from the streamlined top of the probe was fivetimes the probe diameter.

In the wind tunnel described above the maximal obtainable and usable flow velocity depended on the test absorber. With porous absorbers and Helmholtz resonators with relatively smooth surfaces flow velocities of up to 80 m/sec were reached in the test tube; $\lambda/4$-resonators with deep and sharp-edged grooves allowed only for 50 m/sec

2) Measuring devices and methods.

In resting air two cone-loudspeakers on opposite sides of the test tube were used as sound sources for the acoustical signal to be measured. The loudspeakers were driven in push-pull and they had an electrical power input of 8 watts each. For measurements with superimposed flow two pressure chamber loudspeakers with an electrical power input of 200 watts each were used in the same arrangement. Power was supplied from a 1 kW-power amplifier. Cooling was achieved either by the flow itself or by a special blower.

The microphone was connected to a variable heterodyne filter of 6 cps bandwidth. The filter was electrically coupled to the built-in beat frequency oscillator, which was also feeding the power amplifier of the single sources. Thus the signal frequency was always in the center of the pass-band of the filter.

For attenuation measurements the voltage at the filter output was registered on a logarithmic level recorder while the microphone probe was drawn through the test tube. The inclination of the recorded line was evaluated in terms of attenuation per unit length (dB/m).

For frequency analysis the paper-drive of the level recorder was coupled to the frequency drive of the variable filter. The output voltage of the filter was registered while the frequency of the filter was continually varied. Fig. 2 gives a block diagram of the arrangement for the measurement of the phase velocity of the sound wave in the duct.

By comparing the phase of microphone output and generator the wavelength in the duct was determined. Phase comparison measurements in streaming air have to be made with frequency selection to ensure a satisfactory signal to flow noise ratio. The voltage at the output of the heterodyne filter can, however, not be used since here the phase relation is lost in the filter. Therefore a phase-sensitive mixer amplifier a socalled "lock-in amplifier", was built. Its bandwidth can easily be adjusted. Measurements were made with a bandwidth of 1 cps. A third octave filter was used for preselection. The signal amplitude at the measuring input of the lock-in amplifier was controlled mechanically to give constant input voltage. For this purposo a level recorder was connected to the heterodyne filter. The recording system of the level recorder is rigidly connected with the slide contact of the logarithmic input potentiometer and it is moved in such a way that the slide contact is always picking up a constant voltage. With this recording system, recording the signal attenuation, the slide contact of another, identical potentiometer was rigidly connected. This second potentiometer was supplied with the amplified and preselected microphone voltage. Following the recording movements of the attenuation recorder the slide contact of the second potentiometer was always moved in a way to pick up a constant voltage of the signal frequency which was then connected to the measuring input of the lock-in amplifier. The reference voltage for the lock-in amplifier was taken from the generator. The d c.-voltage at the output of the lock-in amplifier was chopped and registered with another level recorder. The dynamic of this arrangement was 45 dB. The phase recorder recorded the square of the a.c.voltage from the chopper; thus the phase differences n π (n = 1, 2, ...) are clearly marked for evaluation. Of course the over-all dynamic is limited by the dynamic of the attenuation recorder as soon as this falls below 45 dB. The limits of the measuring accuracy are given at low frequencies by the number of wavelengths in the duct and at high attenuation values by the number of wavelengths displayed on the attenuation recorder.

In all experiments the frequency was read from a frequency meter calibrated with a quartz frequency standard. The accuracy of the attenuation measurements differs for the different frequencies, flow velocities and absorber types because of the varying dynamic and the varying evaluation accuracy of the inclination of the recorded attenuation lines. The accuracy is generally lower for very small (below 1.5 dB/m) and very large (over 200 dB/m) attenuation. The signal to noise ratio during the attenuation measurements was, if not explicitely stated otherwise, at least 25 dB; it was usually between 35 and 45 dB. The standing wave ratio of the sound pressure caused by reflections at the duct termination was below 4 dB. The cross-talk through the probe tube was 55 dB below the sound level through the lateral holes at the top of the probe. The cross-talk from the measuring room into the duct was below - 45 dB.

Part II

Attenuation with Dissipative Absorbers.

For air-borne sound attenuation mostly porous absorbers or highly damped resonators are used; these are absorbers with predominating dissipative attenuation. The influence of the flow on the propagation attenuation can for these types of absorbers be described as the result of three usually interfering flow effects.

A. Influence of the Flow an the Air-Borne Sound Attenuation.

In order to obtain a general survey over the possible flow effects, we shall derive an expression for the propagation attenuation of the fundamental mode in a duct with absorbing walls and a superimposed air flow for the simple case of a plane wave from a discussion of the energy. With this expression we will be able to recognise and discuss typical changes of the attenuation constant which are caused by the flow.

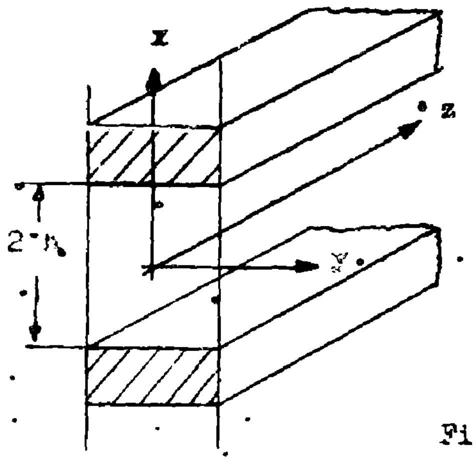

Fig. 3 .

The wave shall propagate in z-direction. The duct has a rectangular cross-section with absorbing surfaces at $x = \pm h$. We look at a section of unity length of an infinitely broad duct in y direction. We allow for lateral distribution of the sound field values in x-direction due to the wall coatings.

Between the sound pressure $\bar{p}_w$ at the wall, the particle velocity $\bar{v}_n$ normal to the wall and the admittance L of the wall we have the following relation

$$\bar{v}_n = \bar{L}\,\bar{p}_w \qquad (1)$$

(the horizontal line ¯ indicates complex values). The real part of the sound power penetrating through a wall section dz is

$$-\,dN = \frac{1}{2}\,(\bar{p}_w \cdot \bar{v}_n^{*} + \bar{p}_w^{*} \cdot \bar{v}_n)\;dz = \mathcal{R}e\,(\bar{L}) \cdot |\bar{p}_w|^2 \cdot dz \qquad (2)$$

(* indicates conjugated imaginaries). With I(x) the sound intensity we obtain for the sound power penetrating through a cross section of the duct with the height h and the width unity

$$N = \int_0^h I(x)\,dx \qquad (3)$$

Whence follows from (2)

$$\frac{dN}{N} = - \frac{\mathcal{R}e(\bar{L})|\bar{p}_w|^2 \; dz}{\int I(x)\, dx} \tag{4}$$

Whence follows

$$N(z) = N_o \exp \left(- \frac{\mathcal{R}e(\bar{L})\; |\bar{p}_w|^2 \; z}{\int I(x)\; dx} \right) \tag{5}$$

We substitute $\alpha = \rho c \cdot \mathcal{R}e(\bar{L})$ in (5), use the well known expression $I = p^2/\rho c$ for a plane wave without flow and obtain

$$N(z) = N_o \; e^{-\alpha z/h} , \tag{5a}$$

which is a generally used approximation for low frequencies and not too high wall admittance ($|p_w| = |p|$), [3, 4]. Equation (5) is valid over a wider range since there the sound pressure distribution is taken into account.

We shall, however, only try to get a survey over the effects of flow on the sound attenuation with the help of equation (5), by studying the behaviour of the single components in the attenuation exponent. We will therefore only deal with the most simple cases.

1) Change of wavelength.

We are to discuss the factor $z/\int I(x)dx$ and therefore write

$$N(z) = N_o \exp \left(- Az/ \int I(x)\; dx \right) \tag{5b}$$

taking A as constant. This expression is compared for a plane wave in a resting medium with the expression for a plane wave in a streaming medium.

In resting medium we have

$$\int_o^h I(x)\; dx = h\; p^2/\rho c = I_o\; h \tag{6}$$

A simple calculation [5] reveals, that the sound intensity of a plane wave in a one-dimensional flow in the direction of the wave normal is equal to

$$I_V = \frac{p^2}{\rho c} (1 + M) = I_0 (1 + M) \tag{7}$$

with the exception of terms of the third order. Here ρ is the density of the resting medium, p the sound pressure, c the adiabatic sound velocity, $M = V/c$ the Mach number of the flow, and I_0 the sound intensity of the plane wave without flow.

Equation (5b) now writes

$$\begin{aligned} N(z) &= N_0 \exp \left(- \frac{A\, z}{I_0 h (1 + M)} \right) \\ &= N_0 \exp (- \alpha_V \cdot z) \end{aligned} \tag{8}$$

with α_V the attenuation per unit length at a flow velocity V. It is obvious that

$$\alpha_V = \frac{\alpha_0}{1 + M} \tag{9}$$

If, however, the attenuation is related to the wavelength λ, i.e.

$$N(z) = N_0\, e^{- \lambda\alpha \frac{z}{\lambda}} = N_0\, e^{- \alpha' \frac{z}{\lambda}} \tag{10}$$

and if the change of the wavelength is simultaneously taken into account

$$\lambda_0 \longrightarrow \lambda_V = \frac{c + V}{f} = \lambda_0 (1 + M) \tag{11}$$

it becomes obvious, that the attenuation constant α' is not affected by the flow

$$\alpha'_V = \alpha'_0 . \tag{12}$$

Thus the attenuation change caused by the flow as expressed in (8) can be attributed to the increase of the sound wavelength for propagation in flow direction (forward propagation: $V, M > 0$) or the decrease of the wavelength for propagation against the flow direction (backward propagation: $V, M < 0$) resp.

The invariance of the attenuation constant related to the free field wavelength is a consequence of our deduction from a plane wave in a duct with a small wall admittance $\bar{L}$. It is obvious that generally the attenuation constant should be related to the duct wavelength λ and that instead of (11) the wavelength changes according to

$$\lambda_v = \lambda_o \left(1 + \frac{V}{u_o} \right) \tag{13}$$

u_o is the phase velocity in a duct with resting air.

If this flow effect should give a complete explanation of the change in attenuation then - as assumed here - the pressure distribution over the cross-section of the duct, the wall conductivity and the phase velocity relative to the medium should remain unaffected by the flow.

2) Change of the pressure distribution.

In expression (5) for the attenuation exponent the pressure $\bar{p}_w$ at the absorber surface is contained. This pressure can be changed by the flow while all other factors remain unchanged.

When a real gas is streaming through a duct the flow velocity is zero at the wall. It is increasing in a boundary layer to the values of the center flow. In any given distance from the wall the phase velocity of the wave is approximately added to the flow velocity at this test point. Except from the curvature of the phase plane in front of a porous absorber already in resting air [6] these phase planes are additionally curved by the velocity gradient of the flow. The sound pressure at the wall will be affected by the flow.

The curvature of the phase planes and of the sound beams by the gradient field of the phase velocity is a typical problem of geometrical acoustics. It is a characteristic for geometrical acoustics that all properties of the material involved undergo a marked change only over distance large compared to the wavelength. With the duct cross-section and frequencies involved in our measurements no important change of the pressure distribution could therefore be expected.

For a duct with sound hard walls Pridmore-Brown [5] has calculated the change of the pressure at the wall, $\tilde{p}_w$, caused by the flow for the fundamental mode propagating in flow direction. The calculation was made for two different velocity gradients: constant gradient and turbulent velocity profile. An approximate solution of the two-dimensional, dissipation free wave equation with superimposed stationary, transversely variable flow was calculated. For constant velocity gradients the solution is valid for low values of the gradient and for turbulent flow profiles it is valid for boundary layer thicknesses large compared to the wavelength.

In both cases an increase of the pressure at the wall over the pressure in the center of the duct is found. The pressure increase is reduced with decreasing Mach number of the center flow and with a decreasing number of wavelengths per boundary layer thickness. If the effective height h of the duct is supposed to be the boundary layer thickness then with a linear velocity profile and Mach number $M = 0.2$ (this was the order of magnitude reached in our measurements) the pressure at the wall exceeds the pressure in the center of the duct by 55 dB for $kh = 20$ and by 10 dB for $kh = 2\pi$ i.e. $\lambda = h$. For a turbulent profile the pressure increase is only 17 dB for $M = 0.2$ and $kh = 20$ instead of 55 dB with the linear profile. The kh-values 2π and 20 are the only values which were calculated by Pridmore-Brown in a tedious numerical procedure. However, by extrapolation to the values present in our investigation $0.08 \leq kh \leq 0.8$ it can be shown that this geometrical-acoustical flow effect does not play a role in our measurements, if not the calculation for a wall with finite wall admittance would lead to essentially different values.

Pridmore-Brown has also calculated the attenuation per unit length as a function of kh from equation (5) for a turbulent flow profile with a center flow of Mach number 0.4 taking into account the change of the wavelength according to the preceding paragraph and under the assumption of only a small wall admittance. This calculation was made for a plane wave propagating in flow direction. It is shown that for $kh > 5$ the focussing effect of the sound wave at the walls exceeds the increase in wavelength: the attenuation is higher than in resting air. For $kh < 5$ the attenuation is smaller than in resting air due to the change in wavelength.

3) Variation of the characteristic properties of the absorber.

It was in the two preceding paragraphs assumed that the characteristic properties of the absorber - in equation (5) this is the wall admittance $\mathfrak{L}$ - are not changed by the flow.

It is, however, known that with many acoustical absorbers the resistance as well as the reactance depend on the amplitude of the sound vibration. For Helmholtz resonators the non-linearity of the resistance could quantitatively be attributed to the occurence of vortices, [7, 8]. These non-linearities are also observed when strong direct flow is superimposed. Like in resonators also in porous absorbers a non-linearity of the wall admittance can be caused by flow. A wall admittance independent on flow with porous absorbers would demand for complete superimposihg of the sound wave and the streaming in the pores of the absorber caused by the pressure variations of the flow. From the experiences made in measurements of the flow resistance of porous material we have to conclude that this linear superposition cannot be expected.

The influence of the flow on the sound attenuation by change of wavelength and focussing of the sound energy is open to calculation. This is not the case for the variation of the characteristic properties of the absorber. We are still lacking experimental results on the change of impedance caused by flow parallel to the surface.

Since in our investigation duct dimensions and frequencies used exclude the geometrical-acoustical effect and since furthermore the effect of changes of wavelength can be eliminated by calculation we can obtain from the results a first information on the effect of flow on the characteristic properties of the absorber.

4) Sound scattering by vortices.

Another possible influence on the sound attenuation in flow ducts is the scattering of sound by vortices in the flow

E.A. Müller and Matschat [9] have made an experimental and analytical investigation of this type of sound scattering. As a characteristic quantity for a given stationary turbulence pattern of the flow the ratio r/λ was found; r vortex radius, λ sound wavelength. Below $r/\lambda \approx 1$ the scattered sound energy is decreasing rapidly with the fifth power of r/λ, for higher values of r/λ it increases with the second power of the ratio. With the small duct cross-section and large wavelengths used in this investigation sound scattering is without influence on the attenuation. A measurement of the attenuation in our duct with sound hard walls revealed no change of the attenuation at different flow velocities within the limits of measuring accuracy. But also for larger values of r/λ the effects will remain small. The scattered energy is increasing, but for increasing r/λ the scattering becomes more and more forward scattering.

5) Interaction of flow effects.

In paragraph 1) the influence of a change of sound wavelength on the attenuation was discussed under the assumption that the phase velocity of the wave remains constant in a coordinate system moving with the flow. In paragraph 2) only the effect of the flow profile on the cross-sectional distribution of the pressure was discussed; not discussed was the influence on the distribution of the phase over the duct cross-section and the resulting phase velocity of the wave in the duct. The propagation of the wave in the gradient field of the flow can be treated like wave propagation in layered media in which the wave would have an average phase velocity. Also the effect of the change of the characteristic properties of the absorber according to paragraph 3) on the change in attenuation according to paragraph 1) is obvious: if the effective wall admittance of the absorber is varied also the phase velocity of a wave propagated along the absorber is generally changed.

A marked effect of the phase distribution in the flow profile on the attenuation in a duct coated with porous absorbers is even to be expected when the velocity gradient of the flow does not produce a change of the sound pressure $\bar{p}_W$ at the wall as it was calculated by Pridmore-Brown [5] for a duct with sound hard walls.

It is well known [6, 10] that an originally plane sound wave propagating along a porous absorber has a curved phase surface in resting air. This is indicated in the upper diagrams of Fig.4. The hatched area be the absorber with air above it. The sound wave shall propagate from left to right. Then the angle φ_0 between the wave front (in resting air) and the absorber surface is according to

$$\mathrm{tg}\,\varphi_0 = \frac{1}{\sigma}\sqrt{\frac{2\varepsilon}{q}}\;\frac{\eta^2+1}{\eta+1} \qquad (14)$$

$$q = r/\omega\rho \qquad \varepsilon = \rho/\rho_0$$

given by the porosity σ, the flow resistance r and the ratio of air density ρ in the material to the density ρ_0 in free space.

The curvature of the phase surface in resting air is a consequence of interference of the sound wave in front of the absorber with the near fields of the compressional wave in the material and with the surface wave at the boundary. In resting air the angle φ_o is generally correlated over the boundary conditions for the three waves with the penetration of sound energy into the absorber so that a smaller angle φ_o stands for a higher penetrability i.e. for a higher propagation attenuation.

Even if we leave out of consideration that a change of the properties of the material by a superimposed flow is possible, the angle between phase plane and absorber surface can be affected by the phase distribution in the flow profile. In the second row of Fig.4 a diagram of the flow profile is given for both directions. The superposition of the phase velocity u of the sound wave and the transverse variable stream veloci V is also indicated. In the case of sound hard boundaries and boundary layer thicknesses large compared to the wavelength the wave fronts of the sound wave are deformed by this superposition into a shape resembling the flow profile. In this case we will also observe the sound pressure distribution discussed in paragraph 2). If the transverse dimensions of the flow profile are smaller than the wavelength, however, the pressure [illegible] constant over the cross section and the phase surface of the wave can no longer be constructed by simple addition of phase velocity u and local flow velocity V; there is, however, still an additional curvature of the wave fronts at the wall caused by interference in the flow profile. This additional curvature of the wave fronts has the same direction as that of the flow profile. By adding the curvature of the wave fronts caused by the flow to that which is already existing in resting air in front of porous absorbers, the resulting angles φ_v are different from one another depending on the sound propagating in or against the flow direction as it is indicated in the third row of Fig.4. This change in angle is not connected with a change of the absorber properties according to equ. (14), such an effect can occur additionally. In any case the front-surface angle φ_v is correlated with the penetration of sound energy into the absorber; although the correlation is different from that in resting air. In the neighborhood of the absorber flow and wave front include the angle φ_v. According to Blokhintsev [11] the sound power averaged over one period penetrating an area parallel to the flow of unit cross-section is given by

$$N(\varphi_v) = \bar{p}\,\bar{\bar{v}}\left(1 + \frac{V}{u}\sin\varphi_v\right)$$

Whence follows that different attenuation characteristics are to be expected with porous absorbers for the two possible propagation directions relative to the flow direction. This dissymmetry is disclosed in the measurements described later. For a quantitative investigation more measurements should be devoted to the sound field and the flow profile at the absorber surface.

2. Attenuation Measurements with Dissipative Absorbers.

As a prototype of [illegible] porous absorber a rock wool layer ([illegible] produced by [illegible] und Hartmann) of 78 mm thickness covered with perforated sheet on the flow side was tested. The perforated sheet was 1 [illegible] thick; the holes had a diameter of 3 mm; the surface area of the holes was 36 % of the entire sheet surface. With the help of a cover board on the rear side the absorber was fastened to one broad side of the duct under moderate pressure. The cross-section of the duct was 33 x 100 mm^2.

In Fig.5a a sketch of the cross-section is given; also given is the attenuation α in dB/m as a function of the frequency in kcps. The flow velocity is taken as a parameter. Positive values of V indicate sound propagation in flow direction, negative values indicate propagation against the flow direction.

During these measurements as well as during the following experiments concerning sound propagation in both directions two pressure chamber systems were used; one at each end of the test duct. Thus attenuation measurements could be carried out in both directions without changing the absorber arrangement.

It was shown in [1] that the scattering of the measured values is caused by bending vibrations of the perforated sheet.

For forward propagation the reduction of attenuation is about equal over the whole frequency range. For V = + 70 m/sec and 1.0 kcps the attenuation is about 40 % lower than the corresponding value for resting air. For backward propagation the attenuation is increasing evenly but not at the same rate. The increase in attenuation at V = - 70 m/sec and 1.0 kcps amounts only to 25 %.

The attenuation curves for the different flow rates are not alike. The curve for V = 0 has a hunch at 1.0 kcps. For this frequency the thickness of the rock wool is equal to a quarter wavelength in the material. For forward propagation this hunch is evened out with increased flow velocities. Considering that the hunch at 1 kcps is the result of a resonance - however weak - this observation agrees with the general experience that for forward propagation especially the resonance attenuation of less marked resonances is reduced with increasing flow velocity (comp. Fig.6). The curves for backward propagation, on the other hand, are much more similar to those measured with resting air. We will have to discuss the different variation of resonance attenuation in connection with the measurements with damped Helmholtz resonators.

According to the remarks made in paragraph A. the variation of the attenuation must partly be caused by the change in wavelength due to the superposition of flow. In order to eliminate this influence the phase velocity u_0 was measured in the duct with resting air. Together with the wavelength λ_0 it is plotted in Fig.6 versus frequency f.

The attenuation α_V' per wavelength λ_V in the duct for a flow velocity V was calculated according to

$$\alpha_V' = \alpha_V \lambda_V = \alpha_V \lambda_o \left(1 + \frac{V}{u_c}\right) \tag{15}$$

from the averaged attenuation curves given in Fig. 5b belonging to the measured values indicated in Fig. 5a. The results are plotted as a function of the frequency in Fig. 7. The heavy drawn-out line is the attenuation per wavelength at V = 0. Measured values for backward propagation are found in the hatched area. The curves for forward propagation are given as thin lines.

The different behaviour of the attenuation for the two propagation directions of the sound is obvious. The attenuation per duct-wavelength is nearly constant for propagation against the flow; the deviation in the hatched areas is at first increasing with the flow rate. It reaches a maximum for V = - 50 m/sec (above 500 cps the curve for V = - 50 m/sec gives the upper limit of the hatched range) and then decreases again. The curve for the attenuation per wavelength at V = - 80 m/sec is identical to that for V = 0 m/sec within the measuring accuracy. For signal propagation in flow direction the deviation of the attenuation per duct wavelength can, according to the considerations in paragraph A, only be explained by flow dependence of the characteristic properties of the absorber or by a change of the wave front to surface angle φ due to the velocities gradient in the flow. It is remarkable that most of the deviation is observed already in the velocity interval from 0 to 30 m/sec while the deviation is changing only relatively little at higher flow velocities.

The dissymmetry of the change in attenuation related to the two different propagation directions is very distinct. This is clearly visible in Fig.8, where the difference $\alpha_V' - \alpha_0'$ of the attenuation per wavelength with and without superimposed air flow at frequencies 0.4, 0.6, and 1.4 kcps is entered as a function of the flow velocity V. For 0.6 kcps values derived from the measuring results in Fig.5a are entered as thin lines since here these differ very much from the approximation curves in Fig. 5b. The dissymmetry can be explained with the change of the angle φ by the flow as was discussed in paragraph A.

We have to mention that by relating the attenuation to the wavelength the other flow effects are falsified: an increase in attenuation with rising flow velocity is partly compensated and a dissymmetry with smaller attenuation in forward direction is overaccentuated. If for example the dependence of the attenuation α on the flow rate V is written like

$$\alpha_V = \frac{\alpha_o'}{\lambda_o(1 + V/u_o)} - g\ (V) \tag{16}$$

with g(V) the additional attenuation reduction by the flow, so that

$$\alpha_V' - \alpha_o' \approx - \lambda_c\ (1 + V/u_o)\ g(V) \tag{17}$$

then it becomes obvious that at $V > 0$ g(V) is multiplied with a factor greater than λ_c and at $V < 0$ with a factor smaller than λ_o. The factors differ the more the greater the amount of V.

Another often used absorber arrangement with predominating dissipative attenuation consists of highly damped Helmholtz resonators. In Fig.9 the attenuation constant α in dB/m for such resonators is displayed as a function of the frequency f. Also given there are the geometrical dimensions of the resonators. A trolitul lattice with 33 mm spacing and 2 mm wall thickness was glued to a lucite plate of 10 mm thickness. The lattice separated the resilient volumina of the single resonators. Precisely centered holes (10 mm diameter) in the lucite plate served as resonator necks. The flow resistance was given by a micropore foil glued to the flow side. The cross-sectional dimensions of the duct were 33 x 99 mm^2.

In Fig.9 again the marked reduction of the resonance attenuation for forward propagation is to be seen. Also shown is the increase of the resonance frequency caused by the flow. These observations were already reported earlier. The shift of the frequency of maximal attenuation towards lower frequencies for sound propagation against the flow (comp. Fig.16 for contr.) is unexpected. In backward propagation measurements the attenuation in resonance is increasing at first with the flow corresponding to the contraction of the wavelength. But then the attenuation is dropping again in spite of the further contraction of the wavelength. The obvious reason is the non-linear increase of the flow resistance in the pores of the mikropore foil in front of the resonator necks.

To eliminate the effect of changes in wavelength caused by the flow the phase velocity was measured in the duct with resting air. Together with the wavelength the results are shown in Fig.6. The dispersion within the range of the resonance frequency is visible. The attenuation per wavelength was then calculated according to equ. 15. This is illustrated in Fig.10. The resonance attenuation is decreasing with increasing flow velocity for both directions of propagation. The increase of the attenuation per wavelength at $V = -40$ m/sec and the considerable shift of the maxima at $V > 0$ was caused by a "wrong" choice of u_0 when reducing according to equ. 15. In order to reduce to the real duct wavelength at the flow velocity V, u_0 had to be that velocity, which, added to V, leads to the phase velocity u_V of the wave in the duct at a flow velocity V. u_0 was determined in resting air; however the flow leads to a change of the resonance frequency and thereby also to a change of the phase velocity relative to a coordinate system moving with the flow and this change is especially noticeable at and around the resonance frequency. But since relating the attenuation to the wavelength should reveal all other flow effects on the attenuation it is justifiable to use u_o for the reduction. According to earlier investigations and to results discussed in part III of this work even a single resonator changes its resonance frequency in flow.

Apart from these anomalies caused by the use of $u_c - V$ instead of u_v for the reduction of the attenuation constant a symmetric change of the attenuation with the flow velocity is seen from Fig.8b. In Fig.8b again the difference between the attenuation constants per wavelength with superimposed flow and in resting air is plotted against the flow velocity. The higher symmetry of the attenuation change compared to Fig.8a inspite of essentially the same structure of the absorber surfaces confirms the explanation given for the dissymmetry of the attenuation change with the porous absorber.

Part III

Sound Amplification with Reactive Absorbers

Sometimes minima of sound attenuation in flow ducts with damped resonance absorbers are observed [2] the frequency of which is changing with the flow velocity. These minima are related to the reactive character of the absorbers; they are more clearly visible and can even cause sound amplification when undamped resonance absorbers, i.e. absorbers with predominating reactive attenuation. are used.

Characteristic for reactive absorbers in resting air is a high resonance attenuation with a small bandwidth. Under the influence of the superimposed flow generally shift of the resonance to higher frequencies, tendency towards self-excitation and occurence of deattenuation or signal amplification resp. are observed. In this part of the work deattenuation and sound amplification caused by flow will be investigated.

A. Measurements with Reactive Absorbers.

In [2] attenuation measurements with undamped Helmholtz resonators were reported. For comparison with other measurements the more important results are shown again in Fig.11. The dimensions may be read from the diagram of the cross-section in Fig.11. In these measurements the sound propagated in flow direction. Measurements with backward propagation will be discussed later. For the investigation reported in the work at hand mor powerful signal sources were available than for the former experiments. This enabled a checking of the amplitude dependence of the amplification.

In Fig.12 attenuation measurements for three different signal levels at a flow velocity V = + 45 m/sec is displayed. No absolute calibration of microphone and filter was carried out. All levels are related to an arbitrary reference level; the reference level is the same for all level values given in this text. The signal level during the earlier measurements given in Fig.11 was about 40 dB. The attenuation with superimposed air flow is according to Fig.12 independent on amplitude except in the amplification range. In the same way the attenuation in resting air is independent on amplitude over the whole frequency range within the measuring accuracy. Fig.13 illustrates the amplitude dependence of the amplification in detail. The level of the flow noise in these measurements was about 6 dB. For signal levels up to 45 dB no systematic change of the amplification with the amplitude was observed within the limits of the measuring accuracy. For higher amplitudes the amplification is reduced; simultaneously the frequency of maximal amplification is changing. For all signal amplitudes the recorded sound pressure as a function of the distance from the sound source (logarithmic scale) is a straight line disturbed by a more or less marked wavyness. Fig.14 and 15 show curves recorded on a level recorder during such measurements. In Fig.14 the signal level is about 40 dB. In Fig.15 the curves show:

a) The sound pressure in resting air, b) the flow noise, c) to f) recordings for a flow velocity V_o + 45 m/sec and different signal levels. Fig.15 is an example of a wavy curve. The wavyness at low signal levels at the entrance of the duct might be caused by a parasitic wave of equal frequency but different phase velocity. The accuracy of the phase measurements in the flow was not sufficient to investigate phase velocity and source of the parasitic wave. The occurence of wavyness in amplification ranges is only sporadic (see Fig.14).

The attenuation was measured with the same resonators also with sound propagation against the flow direction; the results are given in Fig.16. For resting air the curve is drawn out; with superimposed flow only the part above the resonance is given for clearness. Below the resonance an attenuation increase is observed corresponding to the change of the wavelength caused by the flow. In the neighborhood of the resonance this effect is masked by the shift and the reduction of the resonance attenuation. The resonance attenua tion is changes to the same degree and in the same direction as with forward propagation. There is no amplification.

For the flow velocity V = 80 m/sec an area around 2 kcps is hatched. Such areas are observed for all flow rates just before the slightly marked attenuation maxima but they are not indicated seperately for the other flow rates.

In these areas the attenuation cannot be defined unequivocally. There are two distinctly different slopes on the recordings: a steeper one in the neighborhood of the sound source and a flatter one in greater distance. Both of them become steeper with rising signal amplitude. The slope observed in greater distance from the sound source is subject to a much stronger change with amplitude than the sound pressure decrease near the loudspeaker. With increasing signal amplitude also the range with the steeper slope is extending. In these frequency ranges only two statements are defined to a certain extent: The drop of the sound pressure near the sound source, which is relatively independent on amplitude, and the frequency of minimal pressure decrease in a greater distance from the signal source. In Fig.16 the slope of the curves recorded at the far end of the duct is plotted as the lower limit of the hatched area for V = - 80 m/sec (at a signal to noise ratio of 15 dB). Otherwise the slope of the recordings in the neighborhood of the sound source (distance up to 80 cm) was used in Fig.16.

We have discussed the pressure course in these areas in more detail since they are very different from the sound amplification with forward propagation. First of all this is true for the amount of deattenuation. Furthermore for backward propagation the inclination of the pressure records at the farther end of the duct shows a strong dependence on amplitude even for the smallest signal amplitudes, while for forward propagation an amplitude dependence of the amplification is noticed only for signal levels exceeding 45 dB. At last for forward propagation the registered curves can always be approximated by straight lines and we would like to emphasize here that in all other measurements in deattenuation ranges the registered curves were straight lines.

In spite of these discrepancies we will later obtain an important information on the sound amplification mechanism from the frequency of minimal inclination of the pressure drop with backward propagation.

In Fig.16 we have attenuation maxima above the resonance the frequency of which is changing with the flow velocity. In Fig.17 these attenuation maxima for negative V, the attenuation for V = 0, and the amplification for positve V are confronted. For equal absolute amounts of the flow velocity the frequencies of the attenuation and amplification maxima are the same. The attenuation maxima for backward propagation show only a small amplitude dependence; for example at V = - 50 m/sec and 1,7 kcps the attenuation is only changing from 6,3 dB/m for a signal level of 55 dB to 5.5 dB/m for a signal level of 30 dB

In order to investigate the conditions for sound amplification the absorbers were varied.

It is an important experience from the experiments that all dimensions have to be kept constant over the entire length of the duct with utmost care and accuracy. Small deviations in the dimensions of the resonators cause frequency deviations and changes of the wave impedance, which influence and often prevent sound amplification.

It is learnt from the results reported in [2] that the sound amplification is critically dependent on the shape of the flow-side end of the resonator necks. Resonators were built the values of the acoustical elements of which - vibrating mass and resilient volume - were identical with the Helmholtz resonators described above. The necks, however, were given a shape so that a stronger interference with the flow was to be expected (see Fig. 18). The necks consisted of tubes with an inner diameter and a length of 10 mm protruding 4 mm into the flow. In Fig. 18 attenuation curves for sound propagation in forward direction are displayed. The attenuation below and in the neighborhood of the resonance frequency shows the usual behaviour. It is only to be remarked that the resonance frequency shift, which can only be explained by a reduction of the vibrating mass by the flow, is smaller in this case than with resonator necks sitting flush in the wall. The deattenuation at $V = +30$ m/sec is, as expected, greater than with resonators with flush necks; at higher flow velocities it is however, smaller. The deattenuation is more and more reduced by higher flowvelocities, while it remained constant with the resonators shown in Fig. 11.

The reduction of the deattenuation with rising flow velocity is a result of the generally experienced fact, that the sound amplification decreases in the same degree as the flow in the duct becomes highly turbulent over the entire cross-section owing to marked obstacles in the streaming

It is also seen from Fig. 18 that the amplitude dependence of the deattenuation - indicated by the hatched area in Fig. 18 between the measuring values corresponding to a change of the signal level of 10 dB - becomes smaller with rising frequency.

Furthermore the attenuation increase at flow velocity V = + 75 m/sec and a frequency of 1.9 kcps is obvious. This is an example for an experience also made with other absorbers concerning the increase of the attenuation in forward propagation measurements in the neighborhood of the self-excitation frequency of the duct [2]. Otherwise the intensity of the self-excitation is very small with these resonators.

Attenuation measurements in forward propagation were als made with $\lambda/4$ - resonators. In Fig. 19 the results are plotted for a single sided comb-line with equal width for gap and wall. The curve for the attenuation at resting air is drawn out. Else only the measuring points have been plotted. Below the resonance the attenuation increase in the range of the self-excitation is again visible (for V = 35 m/sec the frequency of self-excitation S.E. is entered). If both broad sides of the duct are covered with comb lines (Fig. 20) the measured points with superimposed flow are even nearer (in the hatched area) to the attenuation curve for resting air.

Deattenuation is not found under these conditions. This is again attributed to the high degree of turbulence of the flow caused by the sharp edges at the gaps of the comb line. The turbulence of the flow is also expressed by the maximal obtainable velocities of the flow: 55 m/sec and 40 m/sec resp. instead of 80 m/sec with other absorbers.

In order to reduce turbulence the width of the gaps was reduced to 10 mm leaving the period length of 40 mm unchanged (Fig. 21). Now deattenuation is again observed. Remarkable is the deattenuation at V = + 80 m/sec just below the second resonance (dotted line).

A correlation between deattenuation and shift of the resonance frequency of the absorbers both caused by the flow could be supposed. This supposition could be suggested by the attempt to explain sound amplification caused by the flow as a parametric amplification: pronounced frequency components of the flow noise probably together with the intense self-excitation wave of the duct could - working as so-called pump generator - change the reactance of the resonators periodically.

In the case of parametric amplification suitable frequency components must be found by frequency analysis in the turbulent noise. Self-excitation must be omitted as pump generator, since amplification is found also in velocity ranges without self-excitation. Under conditions of amplification, that is with a flow velocity V ≈ + 45 m/sec and with resonators according to Fig.11 the noise without signal was analysed: white noise was found. Only in a small range around the self-excitation frequency the level was increased by about 5 dB. In a second analysis a signal of 1.4 kcps, that is the frequency of maximal amplification for this flow velocity, was added with a 40 dB level. There was no change compared to the preceding measurements, although with parametric amplification the idler frequency between pump frequency and amplified frequency should be amplified too.

Furthermore resonators were built with a large flow dependent shift of the resonance frequency. In order to develop a suitable resonator a single resonator with constant resilient volume (cross-section 31 x 31 mm^2, depth 24 mm) but with necks of different width and length was fixed to the sound hard duct. The resonator was excited in the resilient volume by a sound source with high acoustical impedance. The sound pressure was measured by a probe microphone with a very thin boring. The resonance curve was registered at different flow velocities in the duct by means of the tunable heterodyne filter. It was experienced, that the relative shift of the resonance frequency is the greater the wider and shorter the necks are. With a length of the neck of 3 mm and a width of 8 mm a frequency shift of 48 % was found at a flow velocity V = 70 m/sec. With an arrangement of such absorbers the attenuation for forward propagation was measured (Fig 22). The resonance frequency shift of the whole group was smaller (37 %) than that of a single resonator (reduction of the vibrating mass per resonator). But the shift is still clearly larger than with resonators according to Fig. 11 (22 %). The relative change of the resonance frequency and of the vibrating mass m of one resonator according to Fig. 22 are plotted against the flow velocity in Fig. 23. The dotted lines below V = 40 m/sec are derived from the behaviour of a single resonator in the flow.

The reactance of the resonators is therefore decidedly non-linear. A sound wave with high amplitude or a distinct flow pulsation can change the reactance of the resonators according to a characteristic like that in Fig. 23 and therefore serve as pump generator. According to the basic principles of parametric amplification frequency of these pulsations has to be different from the signal frequency to be amplified. The pump frequency should therefore be found by frequency analysis. But also with this resonator type no pump generator frequency was to be detected. The sound amplification according to Fig 22 is essentially the same as with the other resonators (Fig. 11) inspite of the different tunability of the reactance. Therefore the sound amplification is no parametric amplification.

The mechanism of parametric amplification has to be excluded since there is no flow pulsation serving as pump generator. But we will see later that the amplification is really caused by certain flow pulsation. It must, however, be emphasized here that the frequency of these pulsations superimposed on the flow is equal to the signal frequency.

B. Explanation of the Sound Amplification.

In the investigation on the influence of flow on air-borne sound attenuation with reactive absorbers an interesting physical phenomenon was met: amplification of an acoustical signal along its propagation path through the duct This sound amplification will be explained on the following pages and also possibilities for a quantitative treatment of the problem will be given.

We will use the analogy with the electromagnetic travelling wave tube as a guiding principle. This includes the advantage that the analogous electromagnetic amplification mechanism is easier to put in words. that the characteristic features of sound amplification will become evident and especially the necessity and fertility of the term "pseudeo sound" introduced by Blokhintsev [11] will be seen The theoretically and formally difficult term pseudo sound will be explained in so much detail as is necessary for the explanation of the amplification The term helps with the conceptual separation of the flow and sound phenomena which, with the exception of the equation of continuity, come from the same basic equations. It represents further more the energy transmitting link between flow and sound field because of its non-linearity. The classical analogy between acoustics and electrodynamics is basing on the fact that corresponding phenomena are treated with the same mathematical means. It will be shown that this is not possible in our case We have therefore got to show under which (most simple) conditions for the flow the basic equations of the flow and sound field permit in principle an amplification of sound. We will find that only a spatial periodicity of the staticnary basic flow is required This result leads us to a phenomenological analogy of the sound amplification and amplification of electromagnetic waves in travelling wave tubes with periodic delay lines. The conception of partial waves on such lines will be deduced and applied to our flow duct. The ambivalence of partial waves as a computational term and as a physical reality will be disclosed. With this we have a basis for the comparison of the measuring results given in paragraph A with the conception developed on these pages. Pronounced similarity of sound amplification with the electrical partial wave amplification will be found. We will understand differences with respect to amplitude dependence of the amplification and the sensitivity of the amplification to turbulence in the basic flow.

1) Electromagnetic travelling wave tube.

The principle of the electromagnetic travelling wave tube will be explained to an extent necessary for the illustration of the sound amplification. For further information the reader is refered to the literature on the subject, for example the monographs by Pierce [12], Kleen [13], and Beck [14]. A comprehensive description with ample literature references is found in R Müller and Stetter [15]

The essential elements of a travelling wave tube are the delay line for the electromagnetic wave to be amplified and the electron beam serving as a guide for the space charge waves The delay line wave is reaching into the discharge space with relatively weak stray fields; the space charge waves in turn induce with their lateral near fields alternating currents in the delay line. Both waves are mutually coupled by means of these fields along the propagation path. The delay line wave will be amplified, if it is so much delayed that it is synchronous with the so-called slow space charge wave In stationary coordinates this wave is moving in beam direction. Relative to the electrons in the electron beam, however, the slow space charge wave is moving backwards. Energy is delivered from the kinetic energy of the electron beam A part of this energy is transmitted to the delay line wave by means of the space charge waves The space charge waves are the energy transmitting link between electron beam and delay line wave. They are suited for this purpose because of their essentially non-linear character; space charge waves are the formal expression for pulsation on the electron beam, which can be described as plasma waves in a stationary coordinate system. They have to be distinguished from the usual electromagnetic waves propagating on the delay line of the travelling wave tube in the first place because of their non-linearity; the electron beam is also guiding electromagnetic waves which in contrast to space charge waves can be linearly superimposed. The other essential difference is given by the form of energy in these two wave types. In the electromagnetic wave energy is oscillating between the electric and the magnetic field energy; in the space charge wave energy is oscillating between electron repulsion and kinetic energy of the moving electron mass. The coupling between these types of waves is effected by the common electric energy. The magnetic field component connected with the motion of electrons can be neglected for beam velocities small compared to the velocity of light. There is therefore no radiation from the space charge waves.

Most of these statements can directly be transfered to sound amplification after introducing the pseudo sound. A difference between travelling wave tube and our duct essential for the quantitative treatment comes from the fact that in the travelling wave tube each of the coupled wave types is clearly assigned to its own guide, while in our problem duct flow as well as sound events take place in the same medium or on the same guide. In the travelling wave tube the line wave is predominately led by the delay line; the energy of the stray fields is small; the space charge wave is limited to the electron beam. Calculating the travelling wave tube mechanism each of the waves can be examined on the respective guide without mutual coupling and than a solution for the whole system can be found by introducing suitable coupling factors. Since Pierce [12, 16] this is the classical calculation method. The same way cannot be used with our flow duct because of the common medium for flow and sound wave to be amplified.

2) Sound and pseudo sound.

The introduction of pseudo sound shall help to solve the difficulty caused by the fact that there is only one medium for flow and sound. There is a two-fold relation to the travelling wave tube analogon: only by the introduction of pseudo sound the similarity between sound amplification and the amplification mechanism in the travelling wave tube is established in basic requirements, and on the other hand the comparison with space charge waves can elucidate the pseudo sound.

Even if we leave aside the statistical flow noise generated by turbulence at the limits of the flow or at the microphone itself, a pressure sensitive microphone in the flow will pick up alternating pressures coming from the flow. For example a monochromatic pulsation v cos ωt be superimposed on a stationary flow V_c the entire flow being

$$V = V_o + v \cos \omega t \qquad (18)$$

then we have a pressure P at the microphohe:

$$P = P_o + A \cdot \varrho \frac{\partial v}{\partial t} + B \cdot \frac{\varrho}{2} V^2 \qquad (19)$$

A and B are form factors depending on size and shape of the sound receiver and on the position of the sound receiving openings on the microphone body [11]. If the receiver is only sensitive to alternating pressure and if the pulsation amplitude v is small compared to the velocity of the flow, the receiver will indicate an alternating pressure

$$p = B \cdot \varrho V_c \cdot v \cos \omega t - A \cdot \varrho \omega \cdot v \sin \omega t \qquad (20)$$

although in this example no sound field was superimposed on the flow. The first term in equation (20) comes from the dynamic pressure; it is quadratic in the flow velocity and therefore produces modulation products with the turbulence components of the flow. In order to reduce the disturbing influence of the turbulence on the received sound pressure the geometry factor B of the pressure microphone is made very small for example by making sound receiving openings at the sides of the receiver. Our microphone was constructed according to these points of view. But the second term in equ.(20) is still remaining. It can by no means be removed since a reduction of A will always result in a reduction of the sound pressure sensitivity of the receiver.

This means that a flow pulsation will always be registered by the sound receiver like a sonic event and that it cannot be seperated by the receiver from a real sound field. Therefore these flow variations are according to D.I.Blokhintsev [11] called pseudo sound. Although pseudo sound is picked up by a receiver in the flow like a real sonic event there are important physical differences between them, and by comparison with the preceding paragraph it is confirmed that these differences are analogous to those existing between electromagnetic waves and space charge waves.

In a stationary coordinate system sound is essentially propagating with sound velocity; pseudo sound propagates with the average flow velocity like electromagnetic waves are propagating with the velocity of light and space charge waves with about the mean electron beam velocity. Sound waves of smaller amplitude are linearly superimposed in resting air as well as in flow; but flow pulsations are essentially nonlinear. For sound the compressibility of the medium plays an important role; one of the energy stores of the sound wave is the potential energy of the compression of the medium. Therefore we observe radiation from a sound wave. For pseudo sound in flow with Mach numbers small compared to one the compressibility is of secondary importance. Therefore no radiation is observed from pseudo sound. The analogy with the electromagnetic case is evident, when "compressibility" is replaced by "magnetic field". Like space charge waves pseudo sound is decaying transversely outside the flow in exponential near fields. The analogy to electromagnetic waves is carried still further by the fact that sound and pseudo sound have one type of energy in common: the kinetic energy of the medium particles. However, the potential energy of pseudo sound is not stored in a change of state of the medium, but is represented by the restoring force of the flow limitations.

Pseudo sound is so important for our problem because in a duct the boundaries of which consist of resonators these resonators are also sound receivers picking up the pseudo sound. Since their mouthpieces are sitting flush in the wall the second term in equ.(20) will prevail. An "elastic" boundary of the flow is given by the oscillating air in and in front of the necks. Or, from another point of view, if the boundary of the flow is thought to fall on the walls of the duct (a partition wall in the flow side of the resonator necks must be added in thought) then these resonators form sources of flow with time dependent yield. The possible excitation of pseudo sound by the oscillating motion of the air in the resonator necks is thereby indicated. On the other hand the resonator is sensitive to pseudo sound. The kinetic energy of the flow pulsation transmitted into the resonator is for a very short time stored as potential energy of the compressed air in the resilient volume. Thus a reversible change of the state of the medium is effected by pseudo sound. At certain phase relations radiation of real sound from the resonators can be excited by pulsation of the pseudo sound. Real sound and pseudo sound of the flow are coupled to one another in the resonators.

The most important function of pseudo sound with respect to sound amplification is given in close analogy to the function of space charge waves in travelling wave tubes. Sound waves of low amplitude are linearly superimposed on the flow. They cannot get energy directly from the flow. Direct energetic interaction with the flow is only found with sound waves of "finite" amplitude. The amplitude dependence of sound amplification in our measurements reveals that we have not to deal with these effects. Pseudo sound, however, can because of its non-linearity draw energy from the flow in analogy to the space charge waves. By coupling with the sound field at the resonators this energy is passed on to the sound wave. Like the space charge wave pseudo sound is the necessary energy transmitter from flow to sound field.

We have explained that by oscillating motions in the resonators pseudo sound can be excited and that on the other hand pseudo sound can be converted into sound at the resonators. A detailed picture cannot be given without first introducing partial waves. Their introduction is suggested by the following analytical investigation.

3) Analytical evidence of sound amplification.

It was already mentioned in paragraph 1) that a mathematical treatment of sound amplification with the methods applied in the case of travelling wave tubes is rather difficult. Inspite of extensive analogy of the basic quantities and inspite of good agreement of the experimental results with the experiences made at the travelling wave tubes, an analytical evidence for sound amplification in flow ducts is required for further comparisons.

The problem is limited and defined to the following question: under which simple conditions is sound amplification possible? We will put up with the formulation of a differential equation of the variables in the duct, which will show the possibilities of amplified waves in the duct. The numerical solution would require investigations of the flow itself, which would exceed the measuring possibilities of the present work.

Our postulated conditions are:

1) Flow and sound phenomena are uni-dimensional and extend in z-direction.

2) The flow $V(t, z)$ is separated into a stationary z-dependent fundamental flow $V_0(z)$ and an alternating flow $v_1(z, t)$, which shall include sonic particle velocity as well as an eventual flow pulsation.

3) Quadratic terms of the alternating variables (index $_1$) are neglected.

4) The alternating variables are proportional to $e^{j\omega t}$.

5) Both stationary terms alone and the complete terms respond to the dissipation-free equation of motion without outer mass forces, and the adiabatic equation of state be valid.

6) The stationary density of mass flow $J_0 = \rho_0 V_0$ be free of sources.

Under these conditions and for simplicity under the assumption $M(z) = V_0(z)/c \ll 1$ we have the following equation for the alternating flow density J_1 (the deduction is found in Appendix I)

$$\frac{\partial^2 J_1}{\partial z^2} - 2\left(jkM + M\frac{dM}{dz}\right)\frac{\partial J_1}{\partial z} + \left(k^2 - 2jk\frac{dM}{dz}\right) \cdot J_1 = 0. \qquad (21)$$

The alternating pressure p_1 is realted to the alternating flow density J_1 by

$$p_1 = \frac{c^2}{j\omega} \cdot \frac{\partial J_1}{\partial z} \qquad (22)$$

(21) is a second order differential equation with variable coefficients. It suggests itself, to set up a periodicity of the fundamental flow $V_0(z)$ and thereby also of $M(z)$ in z. The magnitude of the variation and the accurate course cannot immediately be predicted. At least we know, that there are solutions of equ.(21) for periodically oscillating M and that these solutions increase exponentially with z, thus representing amplified waves.

4) Partial waves.

We have above stressed the importance of spatial periodic inhomogeneities of the boundaries of the duct. Therefore we have to investigate sound propagation in ducts with periodic boundary conditions. The influence of the periodicity of the boundary conditions can very clearly be explained with the occurence of partial waves.

The fact that partial waves generally have a smaller phase velocity than the fundamental wave leads to a further extensions of the analogy between flow duct and travelling wave tube. It is known from travelling wave tubes that the coupling delayed wave is not necessarily the fundamental wave but that it can also be one of the partial waves. In our duct the fundamental wave has a phase velocity u_o of about 400 m/sec. This fast wave cannot interfere with the flow with velocities of up to 80 m/sec. Only partial wave amplification is possible.

In a work by R.Müller [17] partial waves are interpreted as components of a spatial Fourier synthesis of the wave spatially distorted in the inhomogeneous flow duct. In Appendix II existence and properties of partial waves for sound propagation in periodic ducts with resting air are deduced as a solution of the boundary value problem. In the corresponding problem treated for streaming medium in Appendix III the same deduction is found.

The definition of partial waves in resting air follows from the presentation of a sound field quantity F resulting from a boundary value problem:

$$F(x, y, z, t) = \sum_n A_n (x,y) \cdot e^{-j\beta_n z} \cdot e^{j\omega t}$$

$$= e^{j(\omega t - \beta_o z)} \sum_n A_n (x,y) \cdot e^{-j \frac{2\pi n}{L} z} \qquad (23)$$

The addends are the partial waves; $A_n(x\ y)$ are their amplitudes (complex and dependent on the transverse coordinates). β_n the phase constants with

$$\beta_n = \beta_c + \frac{2\pi n}{L} \tag{24}$$

β_c is the phase constant of the fundamental wave. The summation index runs from $-\infty$ to $+\infty$. L is the spatial length of period of the duct boundaries.

Obviously only the sum of all partial waves satisfies the boundary conditions. The mutual ratios of the partial wave amplitudes and of the fundamental wave amplitude are only determined by the geometric dimensions and the boundary conditions of the duct; if the latter are constant over the entire length of the duct also these amplitude ratios are constant. Therefore all partial waves have the same attenuation constant as the fundamental wave. The amplitude A_n of the partial waves are generally rapidly decreasing with increasing absolute amount of the partial wave index n. In most cases only the first two partial waves with $n = \pm 1$ are of importance. Furthermore the amplitudes of the partial waves are mostly decreasing exponentially from the inhomogeneous boundary to the center of the duct. The decrease is the steeper the higher $|n|$. Partial waves have a noticeable intensity only in the neighborhood of the inhomogeneous duct boundary. For many possible shapes of the boundary A_{-1} is smaller than A_{+1}.

It is emphasized that all partial waves have the same frequency as the fundamental wave.

Another essential property of the partial waves follows from equ.(24). According to this equation the phase velocity of the n-th wave is

$$u_n = \omega/\beta_n = \frac{u_o}{1 + nu_o/fL} = \frac{u_c}{1 + n\lambda_o/L} \tag{25}$$

with u_o phase velocity and λ_o wavelength of the fundamental wave in the duct. The group velocities v_{gn} of all partial waves are equal to v_{go}:

$$v_{gn} = \frac{1}{dB_n/d\omega} = \frac{1}{dB_o/d\omega} = v_{go} \,. \qquad (26)$$

We see from equ.(25) that for $L \ll \lambda_o$ - this is the case with most delay lines and also in our measurements - the amount of u_n is always smaller than u_o. Because of the constancy of the amplitude ratios of all partial waves also a partial wave can interfere with the electron beam in the travelling wave tube: the energy taken up by this wave is distributed over all other partial waves. The advantage of partial wave amplification is the low beam velocity; the disadvantage is the smallness and the spatial limitation of the partial waves to the boundary region.

That much was to be said about the properties of partial waves in resting air. This case corresponds in the electromagnetic case to a delay line without electron beam. In the case of the electromagnetic delay line the investigation of partial waves would at this point have come to an end. In that case the propagation on the delay line is nearly not affected by the electron beam. It might be assumed that in our inhomogeneous duct the fundamental wave together with all partial waves is simply superimposed to the flow. Synchronism between flow and a partial wave would not be possible in that case.

Treated in Appendix III are the partial wave solution of the boundary value problem with superimposed flow and the question whether, and if so under which conditions, in the inhomogeneous duct with superimposed flow there exists a partial wave with such a phase velocity u_n, that $u_n = V$. The resulting condition says that the flow velocity $V = u_n$ is given by equation (25) with u_o and λ_c being phase velocity and wavelength of the fundamental wave with superimposed flow. For better marking we write $u_{o,V}$ and $\lambda_{c,V}$. Leaving the change of the phase velocity," caused by the change of the characteristic resonator properties due to flow, out of consideration, but writing simply $u_{o,o} + V$ ($u_{o,o}$ phase velocity of the fundamental wave in the duct with resting air) then the condition writes

$$V = \frac{1}{2} u_{o,o} \left(-1 + \sqrt{1 + 4 \frac{L}{n \cdot \lambda_{o,o}}} \right) \quad . \qquad (27)$$

This can with certainty be fulfilled. It results that also in streaming air partial waves interacting with the flow are possible.

5) Partial wave amplification.

After proving the existence of partial waves, which can interfere with the flow, only the energy exchange mechanism remains to be explained. For illustration comparison is made with the travelling wave tube.

Usually the travelling wave tube amplification with the fundamental wave and a homogeneous delay line is explained as follows: if wave and electron beam are approximately synchronous at the beginning of the common travelling path of the electrons the latter are sorted out so that they are concentrated at those parts of the electromagnetic field where the electrons are subjected to a retarding field. The sorting-out is combined with a transfer of energy from the wave to the electrons. As soon as an electron concentration in proper phase relation is established the electrons are moving against the retarding field and by release of energy from the kinetic energy of the electron beam an electromagnetic field on the delay line is induced. This field intensifies the retarding field which in turn brings about a better sorting-out of the electrons in proper phase relation. The electron beam is continually releasing energy over to the wave field along the entire length of their common path.

Usually the same explanation of the amplification is used, when the coupling wave is a partial wave on an inhomogeneous delay line. This is formally correct and the calculation of a travelling wave tube according to this conception leads to quantitatively correct results. This implies that there is a continual energy exchange between partial wave and electron beam over the entire path length.

From a consideration of the field pattern in a tube with an inhomogeneous delay line other conclusions must be drawn. The electrical field strength at the gaps of a comb line is schematically given in Fig.24 for a wave with the fundamental wavelength λ_o. Longitudinal components suitable for the interaction with electrons are almost only present before these gaps. In the second row of Fig.24 the distribution of longitudinal components in a certain plane in front of the delay line is plotted for an instant t_1. The same is repeated in the third row for a somewhat later instant t_2. Obviously the partial waves are interfering jointly and with the fundamental wave so that the longitudinal field component at the walls between the gaps is zero for every instant.

It is therefore to be expected that the energy exchange is limited to the space before the gaps, this would also agree with the conception that the different electron densities could induce electric fields only in the capacities of the gaps.

The condition (25) for synchronism between partial wave and electron beam may also be deduced from the conception of local energy exchange. According to Fig.24 the electron beam is subject to a twofold periodicity of the boundary conditons: the time-constant periodicity of the duct boundaries given by the delay line with the period L and the time-dependent periodicity given by the field pattern of the wave with the period λ_c. By interaction of these two phenomena the electrons are not influenced by the wave in the space in front of the walls between the gaps. Energy exchange is rendered possible if a certain pack of electrons is meeting the wave in the slots always with the same phase relation. In the space in front of the partition walls the pack can be overtaken by any number of waves n without being influenced. The running time $t_V = L/V$ of the electrons with the velocity V over a length of period L has to be equal to the running time t_{uo} of the wave with the phase velocity u_o over the same path plus a number of n periods $T = 1/f$ of the wave:

$$L/V = t_V = t_{u_o} + {}_uT = L/u_o + n/f. \qquad (28)$$

Whence follows

$$V = \frac{u_o}{1 + \frac{nu_o}{fL}} = u_n \quad ,$$

which is the already known condition.

This deduction is more intuitive; the conception of partial waves has a greater significance. The apparent contradiction of these two ways of consideration with respect to the localisation of the energy exchange reveals the superiority of the partial wave conception. It is thereby possible to explain how the initial electron concentrations are effected. Similar to the Fourier analysis of a function of time the partial wave conception is senseless if pathes smaller than the period length are considered. And like the Fourier components of a function of time have a physical meaning only when applied to a resonance system, the partial waves are effective only if they are synchronously travelling with an influencable system over a longer path. A resonator used as probe for overtones gives an effect only after a building-up time. And correspondingly the electrons used as indicators for partial waves can fully interfere with the partial wave only after a certain common travelling path.

We have discussed the partial wave amplification in the travelling wave tube in so much detail. since here the phenomena are much easier to be explained. Now a direct application to our flow duct is made easy. If in Fig.24 the partition walls are replaced by the slits of resonators, an approximate picture of the particle velocities in the free duct cross-section is obtained. The curves in the second and third row of this figure now represent the course of the sound pressure or the longitudinal particle velocity. By replacing u_c by $u_{o,v} = u_{o,c} + V$ in equ.(28) we obtain from the condition that a certain medium element has to meet the sound wave at the mouth piece of a resonator always in the same phase the conditional equation (27). Now it becomes obvious that the conditional equation (25) for streaming air with the curious requirement, that the partial wave should be resting in the moving medium. says that under the conditions given by equation (25) the fluctuation of the boundary due to the oscillation of the air in the necks of the resonators comes to rest for an observer moving with the flow. (Constancy is obtained for a moving observer only if the amplitude of the n-th partial wave is exceeding all others; if this is not the case only a certain part of the fluctuation is turning constant). If the condition (25) is discussed from the point of view that the flow is carrying along a wave "resting" in the flow, the real meaning of this wave is immediately made clear. it represents a pulsation of the flow. that is pseudo sound. Here again the peculiarity of our flow duct becomes obvious, where guided wave and beam pulsation coincide spatially and where both of them are transmitted by the same medium. Considering the remarks made in paragraph 2) on the coupling between sound and pseudo sound at the resonators and considering further the peculiarities of the flow duct the application of the above deduction for the partial wave amplification in travelling wave tubes with localised energy exchange on the sound amplification is made clear if the electron concentration is interpreted as a flow pulsation. The occurence of such flow pulsations can also in this case only be explained with the help of partial wave excited by the sound field. Representing a stationary pressure and velocity pattern in the medium it can only be built up by the relatively weak sound field on a longer common travelling path. In analogy to the travelling wave tube the medium density in the retarding phase of the sound pressure of the coupling wave is increased above the mean density.

With the help of pseudo sound, representing according to Blokhintsev a periodic flow pulsation, sound amplification can be described in analogy to the travelling wave tube. For an analytical description of the phenomenon the oscillation of the air in the resonator necks could be replaced by surface sources with time-dependent, mutually phase correlated yield. Sound pressure could then be calculated in the duct according to the Kirchhoff's integral generalised for flow.

C. Comparison with Experimental Results.

Now the measuring results with respect to sound amplification obtained with reactive absorbers will be compared with the above explanation. Since the experimental investigation within the scope of this work was limited to acoustical methods, only those informations can be used for comparison which were obtained from measurements of attenuation and phase velocity. At first the results of the more accurate measurements with resonators illustrated in Fig.11 will be discussed. The results for the other structures discussed in section A will be given in short.

Fig.25 illustrates measurements of the phase velocity in the duct according to Fig.11 in resting air and with a flow velocity V ≈ 45 m/sec in sound propagation direction (the measurements were carried out with an apparatus more simple than that described in Part I; this explains for the scattering of the measuring points). Also entered is the attenuation constant α in dB/m obtained under the same conditions. From these phase velocities of the fundamental wave were calculated according to equ.(25) the phase velocities u_{+1} and u_{-1} of the partial waves with $n = \pm 1$ and according to equ.(27) the flow velocity V for which synchronism with these partial waves is possible. In Fig.26 the curves marked u_{+1} and u_{-1} were calculated according to equation (25) replacing u_o by the phase velocity $u_{o,c}$ in resting air: The curves marked V_v and $-V_r$ were calculated from the more accurate equation (27) also using $u_{o,o}$. For forward propagation both curves are nearly coinciding. This is caused by the high values of $\lambda o/L$ in our measurements for which equs.(25) and (27) are nearly equal. (If the phase velocities of the first partial waves are calculated according to equ.(25) using $u_{o,v}$ the deviations from the curves V_v and $-V_r$ are very small). The difference between $-V_r$ and $-u_{-1}$ becomes very large according to these calculations, if $\lambda o,o/4$ is approaching the period length L. In this case a certain medium element is not able to meet the sound wave at the next resonator in the same phase relation without meeting a whole sound wave in the interval. Synchronism is now possible only with the partial wave of the next higher order.

In Fig.26 pairs of values of frequency and flow velocity for maximal deattenuation with propagation of the sound in flow direction (circles) and the corresponding values (triangles) for sound propagation against the flow direction are entered. These points respond better to the curves for $u_{\pm 1}$ than to those for $V_{r,v}$. For backward propagation this is explained by the fact that the partial wave with $n = -1$ is essentially limited to the immediate neighborhood of the wall. A calculation for slit resonators reveals that this spatial limitation is more marked for $n = -1$ than for $n = +1$. Another contribution - predominately effective for forward propagation - to this behaviour is given by the fact [12] that for lines with attenuation in the uncoupled state, i.e. here in resting air, the maximum of amplification is with rising attenuation moving to flow velocities smaller than the phase velocity of the coupling wave. Therefore the points just above the resonance, that is in a range with high line attenuation, are noticeably below the curve for synchronism and approaching this at higher frequencies with decreasing attenuation.

The pronounced reduction of the amplification caused by the line attenuation, well known from the travelling wave tube, is visible in all measurements. Especially detrimentous is the line attenuation for the oscillatory power of backward wave oscillators. The deattenuation with backward measurements is effected by such regenerative amplification mechanism. But its not very distinct behaviour is also evident, since a certain medium element has a much shorter time of stay in front of the resonators in the correct phase relation than with forward propagation.

The wavyness of the sound pressure at the entrance of the duct found in attenuation measurements at certain frequencies is in agreement with the theory of the travelling wave tube. According to this theory four waves exist in the duct under amplification conditions. One of them is propagating against the flow the other three in flow direction. One of these three waves is amplified and another is attenuated. Under certain conditions of flow velocity and frequency they have nearly, but not quite the same phase velocity; this gives rise to a wavyness of the sound pressure in the entrance of the duct. The wavyness is also a consequence of line attenuation; on dissipation free lines the phase velocities are accurately the same over the entire amplification range and the wavyness is therefore disappearing. Wavyness was mainly found in the first amplification ranges above the resonance.

In the following pictures pairs of values of frequency and flow velocity for maximal deattenuation in forward propagation are compared with the phase velocity of the first partial wave for those absorbers from section A, which show deattenuation.

In Fig.27 phase velocity and wavelength of the fundamental wave in a duct according to Fig.18 with resonators with protruding necks are entered. The measurements were made in resting air and at a flow velocity V = + 40 m/sec. It is obvious that the phase velocity in streaming air is not a linear combination of phase velocity in resting air and flow velocity. At 1.55 kcps the difference between these two velocities is only 20 m/sec, although here the resonance shift should produce a difference greater than 40 m/sec. This is a consequence of the influence of the deattenuation on the dispersion. Fig.28 represents the comparison of points of maximal amplification with the partial wave velocity, calculated with $u_{o,40}$. The same is illustrated in Fig.29 for the comb lines according to Figs. 19 and 21. Here the phase velocity u_c in equation (25) was that measured in resting air. This may be done since the superposition of flow has very little effect on the partial wave velocity. Even in Fig.30 with the resonators according to Fig.22 with the large flow dependent shift of the resonance frequency there is not much difference between the phase velocities of the partial wave in resting air and at V = + 55 m/sec in the frequency range indicated in the figure.

From these figures good agreement of flow velocity and frequency of maximal deattenuation with the dispersion curve of the first partial wave is seen. This result is the main experimental confirmation for the correctness of the conception of the sound amplification mechanism as it was deduced and developed above. Several other details found during the course of the measurements confirm the close relation between travelling wave tube and sound amplification. Some of these details were described above, others were not distinct enough for a discussion in this work or were not closely examined. So, for example, a slight increase of the attenuation just above the amplification range can be seen in the attenuation curves of Figs. 11 and 12. According to the dispersion diagram of the partial wave the partial wave is propagating faster than the flow at the corresponding frequencies above the amplification range. Like in a linear accelerator energy is exchanged from the sound wave to the flow

In the same way the attenuation maxima for backward propagation found in Figs. 16 and 17 just above the ranges of regenerative backward wave amplification (not very distinct here) are to be explained. For a correct comparison of these attenuation increases it has to be kept in mind, that due to the change of wavelength caused by the flow (as discussed in Part II A 1) the attenuation for forward propagation should be lower than that for resting air. and that the attenuation for backward propagation should be higher than for resting air.

Also in the turbulence and amplitude dependence of the sound amplification similarities to the travelling wave tube are found, although here the different types of wave guide which have several times been mentioned before should have a considerable influence. The sound wave is propagating in the flow itself and is therefore directly influenced by the turbulence, whereas in the travelling wave tube "turbulence" of the electron beam has a direct influence only on the space charge waves. In fact, the phenomena in the electron beam of a travelling wave tube operated at high amplitudes and high space charge densities closely resemble a turbulence [18]: by overtaking of electrons in the beam very unregular density patterns are occuring along the travelling path; the temporal course of the electron density at a certain point of the delay line in some distance from the electron source displays a great number of overtones; by this "turbulence" electron packs are disintegrated. As soon as more electrons are transported into the accelerating phase of the wave by these overtaking processes than into the retarding phase the signal amplitude is reduced and the amplification goes over into attenuation. This mechanism may without further difficulties be applied to the turbulence sensitivity of the sound amplification. According to our conception the sound amplification is based on the fact that a pulsation in the right phase relation is built up in the flow, that this phase relation is kept up and that the pulsation is amplified over the travelling path. These pulsations, being mere flow events, are by velocity fluctuations in turbulent flow restrained from keeping up the proper phase relation if not already in the beginning the establishment of this phase relation was prevented altogether. The main difference to the travelling wave tube is that in our case turbulence is to a great extent independent on the sound wave, whereas in the travelling wave tube turbulence is produced by overtaking effects in the electron beam produced by high signal amplitudes. Better agreement may be expected with respect to the amplitude dependence of the amplification. As in the duct also in the travelling wave tube constant amplification is observed over a wide range of lower amplitudes (in [18] this range is given with 20 to 30 dB: comp. Fig.13); for increasing amplitudes the amplification is rapidly decreasing and changing into attenuation under certain conditions. For the amplitude dependence of the sound amplification we have to consider that it occurs (acc. to Fig 13) at those amplitudes (concluded from an estimation of the sound power input of the duct) for which the Helmholtz resonators alone become non-linear because of the high sound levels in the resonator necks. This is underlined by the smaller amplitude dependence of the amplification at higher frequencies: for constant sound pressure level the non-linearity of the Helmholtz resonators is decreasing with increasing frequency [19].

However, if we discuss turbulence and amplitude dependence of the sound amplification we leave the implicit condition for our explanation of the amplification mechanism, namely the linearization of the problem, that means small amplitudes.

D. Conclusions.

We have given an intuitive explanation for sound amplification as it occurs in spatially inhomogeneous flow ducts coated with reactive absorbers. This explanation given in close analogy to the amplification mechanism of the travelling wave tube permits the interpretation of a number of measuring results found in relation with the sound amplification and serves as a guiding principle for further investigations. A quantitative treatment of a linearized theory would require a calculatory expenditure unusual in acoustical investigations. It is questionable whether it would lead to better results than the qualitative description. A statement concerning the amount of amplification would be important. It is, however, felt that this depends to a great extent on the shape of the resonators and their necks and this very condition is not easily introduced into a mathematical formulation. The theory will also be unable to consider turbulence in the flow and at the resonator necks It is, however, a matter of principal importance if the results of a quantitative investigation could be compared with the results reported in this work. The solutions indicated for such treatment in this work promise, after suitable experimental investigations on the flow at the resonators, a simple calculation.

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References

[1] Mechel, Fr., Diplomarbeit, carried out in the III.Physikalischen Institut der Universität Göttingen, 1957, unpublished.

[2] Meyer, E.; Mechel, Fr.; Kurtze, G. J.Acoust.Soc.Am. 30 (1958) 165 Experiments on the Influence of Flow on Sound Attenuation in Absorbing Ducts

[3] Sivian, L.I., J.Acoust. Soc.Am. 9 (1937) 135 Sound Propagation in Ducts Lined with Absorbing Materials.

[4] Cremer, L., Acustica 3 (1953) 250 Theorie der Luftschall-Dämpfung im Rechteckkanal mit schluckender Wand und das sich dabei ergebende höchste Dämpfungsmass.

[5] Pridmore-Brown, D.C., J. Fluid Mech. 6 (1958) 393 Sound Propagation in a Fluid Flowing Through an Attenuating Duct.

[6] Schuster, K., Akust. Zs. 4 (1939) 335 Zur Schallausbreitung längs poröser Stoffe.

[7] Bolt, R.H.; Labate, S.; Ingard, U., J. Acoust. Soc. Am. 21 (1949) 94 Acoustic Reactance of Small Circular Orifices.

[8] Barthel, F., Frequenz 12 (1958) 72 Untersuchungen über nichtlineare Helmholtzresonatoren.

[9] Müller, A.E.; Matschat, K.R., Technical Report, 1959, Max-Planck-Institut für Strömungsforschung, Göttingen. The Scattering of Sound by a Single Vortex and by Turbulence.

[10] Meyer, E.; Karmann, R.W., Acustica 1 (1951) 130 Die Schwingung der Luftteilchen in der Nähe einer schallabsorbierenden Wand.

[11] Blokhintsev, D.I., Acoustics of a Nonhomogeneous Moving Medium.
Original: 1946, Leningrad
Translation: NACA TM 1399 (1956)

[12] Pierce, J.R., Travelling-Wave Tubes.
1950, D.van Nostrand, New York

[13] Kleen, W., Einführung in die Mikrowellen-Elektronik.
1952. S.Hirzel, Stuttgart

[14] Beck, A.H.W., Space-Charge Waves and Slow Electromagnetic Waves.
1958, Pergamon Press, London

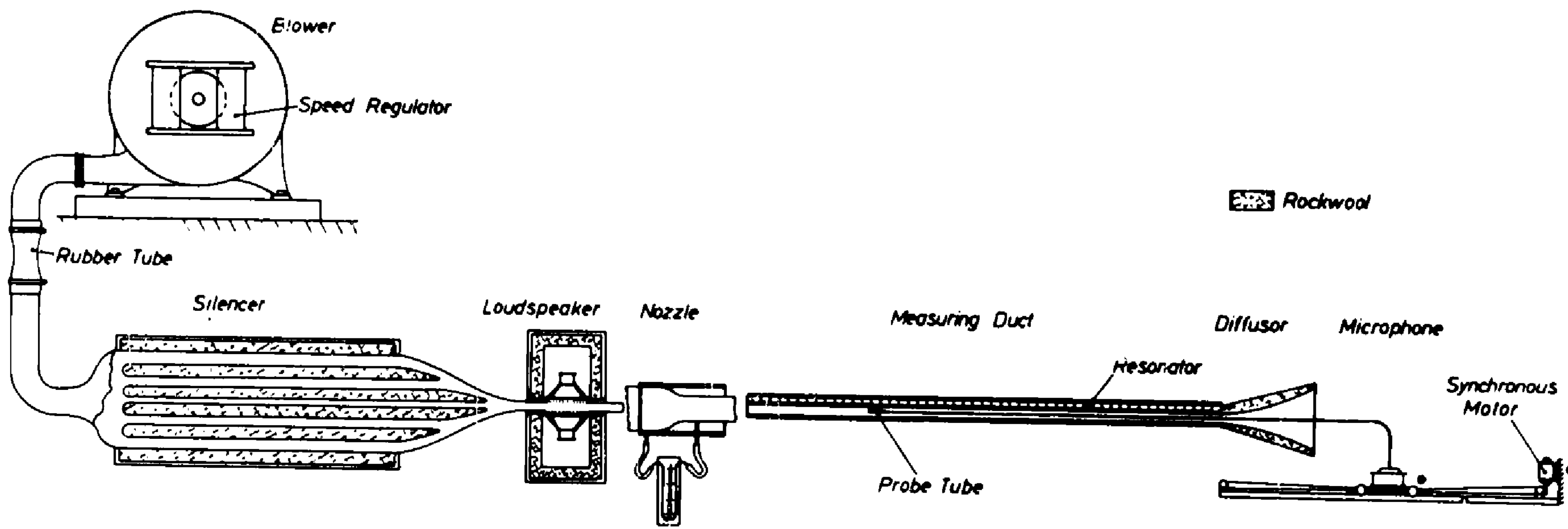
[15] Müller, R.; Stetter, W., Elektron. Rundschau 11 (1957) p.168, 206, 242, 268, 367
Der Stand der Entwicklung und die Wirkungsweise von Mikrowellenröhren.

[16] Pierce, J.R., J.Appl.Phys. 25 (1954) 179
Coupling of Modes of Propagation.

[17] Müller, R., Arch.Elektr.Übertrag.10(1956)505
Teilwellen in Elektronenströmungen.

[18] Tien, P.K.; Walker, L.R.; Wolontis, V.M., Proc.Inst.Radio Engrs.43(1955)260
A Large Signal Theory of Travelling-Wave Amplifiers.

[19] Ingard, U.; Labate, S., J.Acoust.Soc.Am. 22 (1950) 211
Acoustic Circulation Effects and the Nonlinear Impedance of Orifices



Experimental Set-up for Measuring Sound Absorbing Ducts with Air Stream.

Fig.1: Schematic view of the wind tunnel.

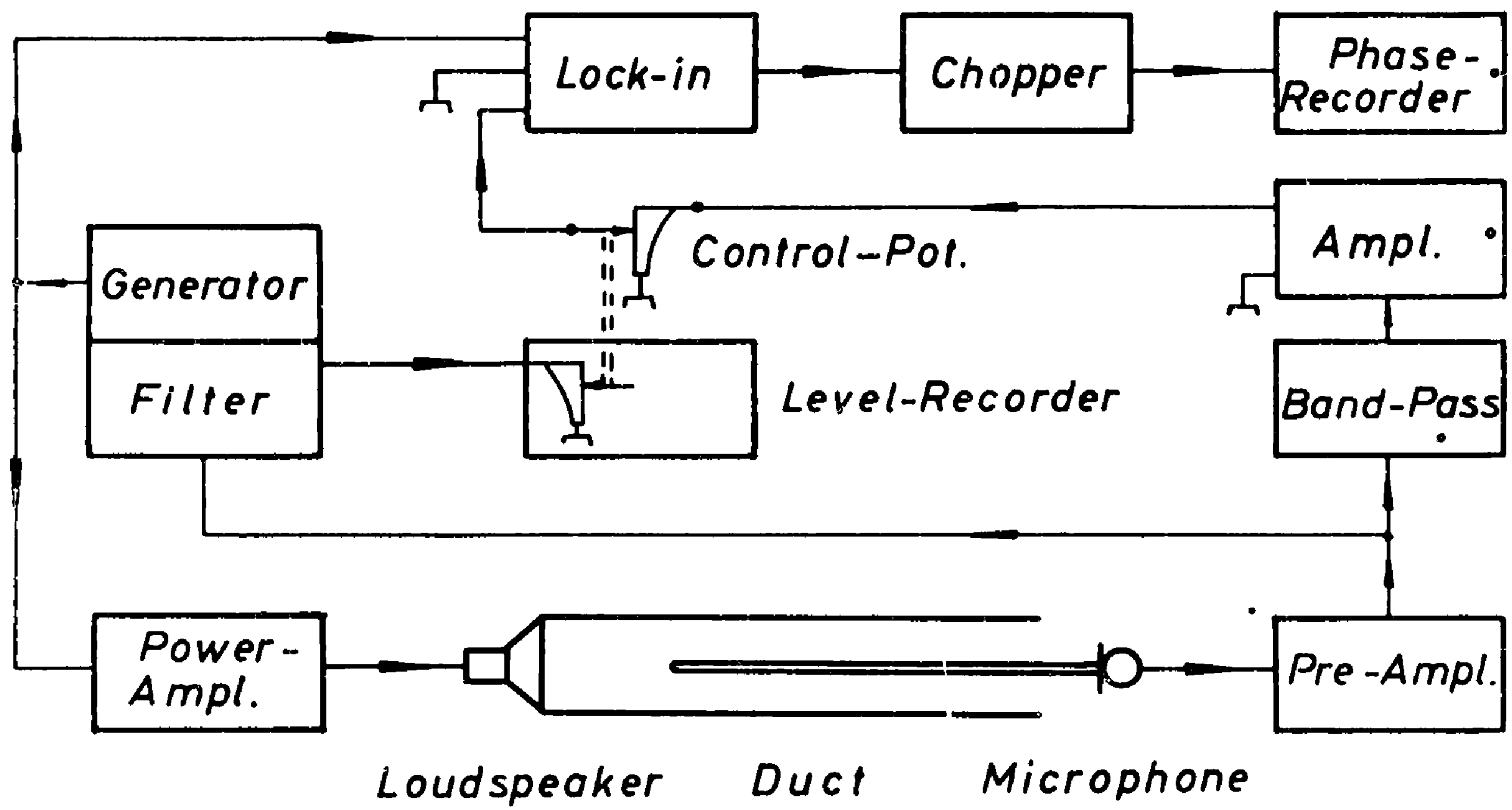


Fig. 2: Block diagram for phase measurements

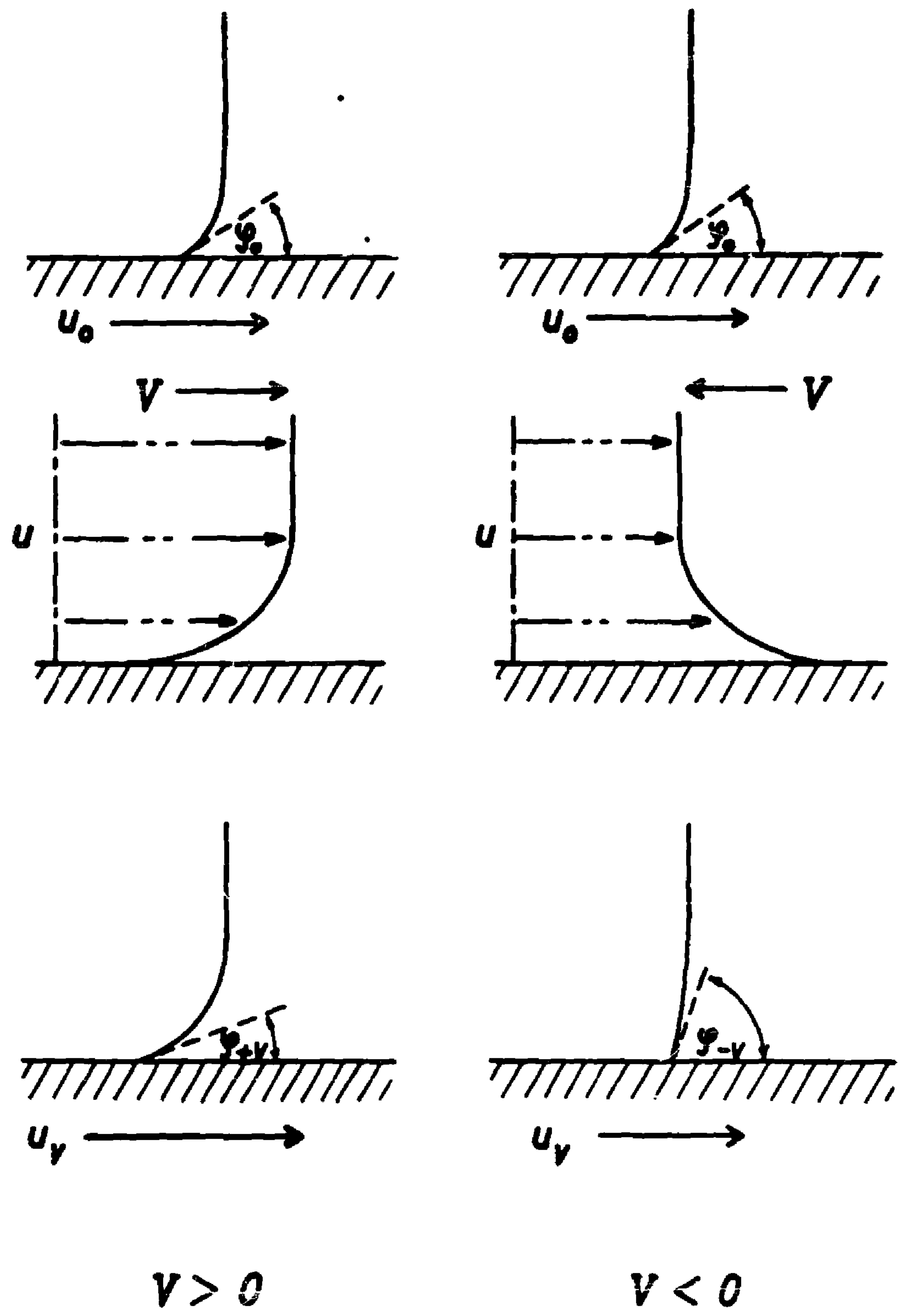


Fig.4: Curvature of wave front in front of porous absorbers

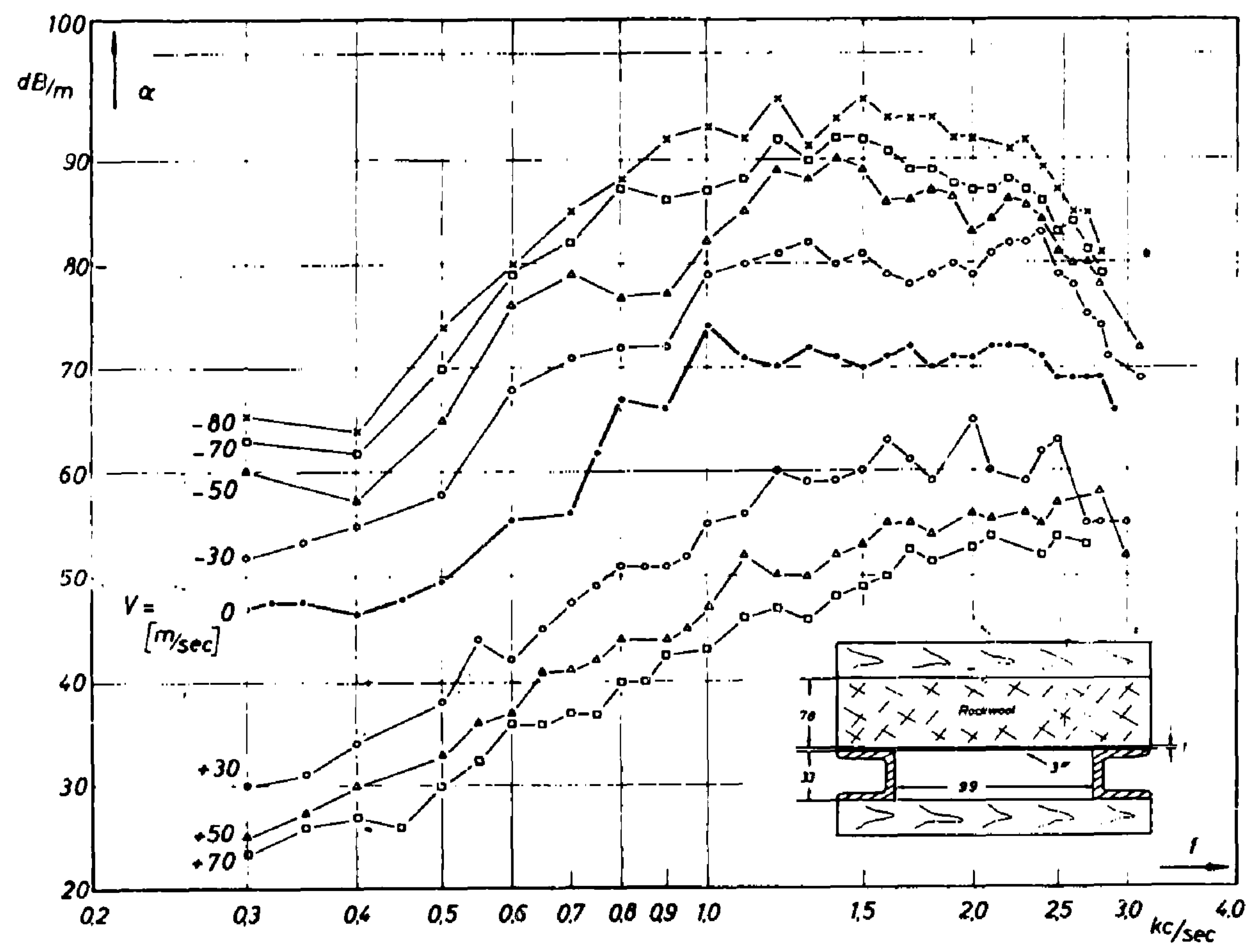


Fig.5a: Attenuation α versus frequency f of a rock wool absorber with perforated cover. Parameter: flow velocity V.

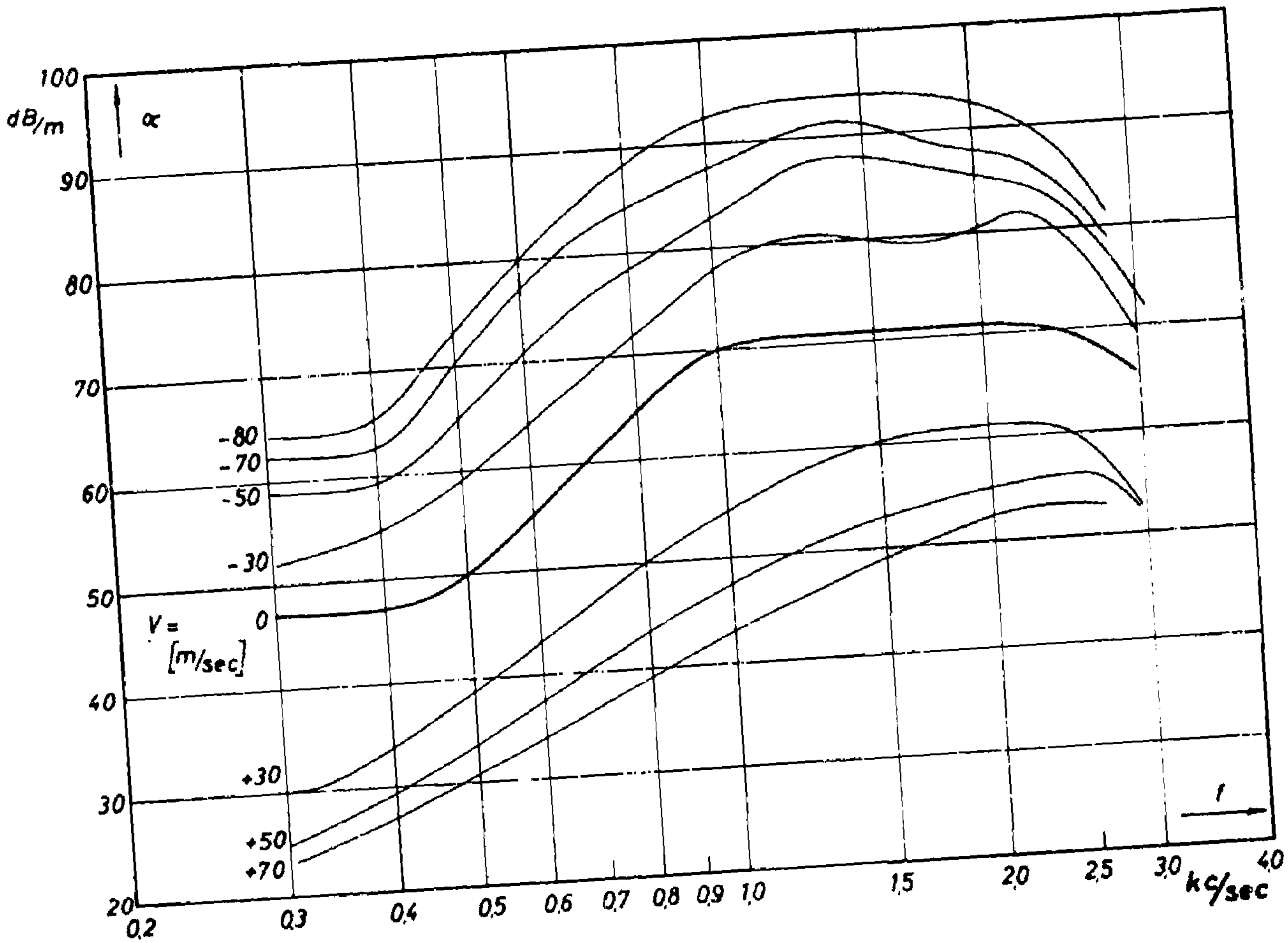


Fig. 5b: Mean values of attenuation taken from Fig. 5a.

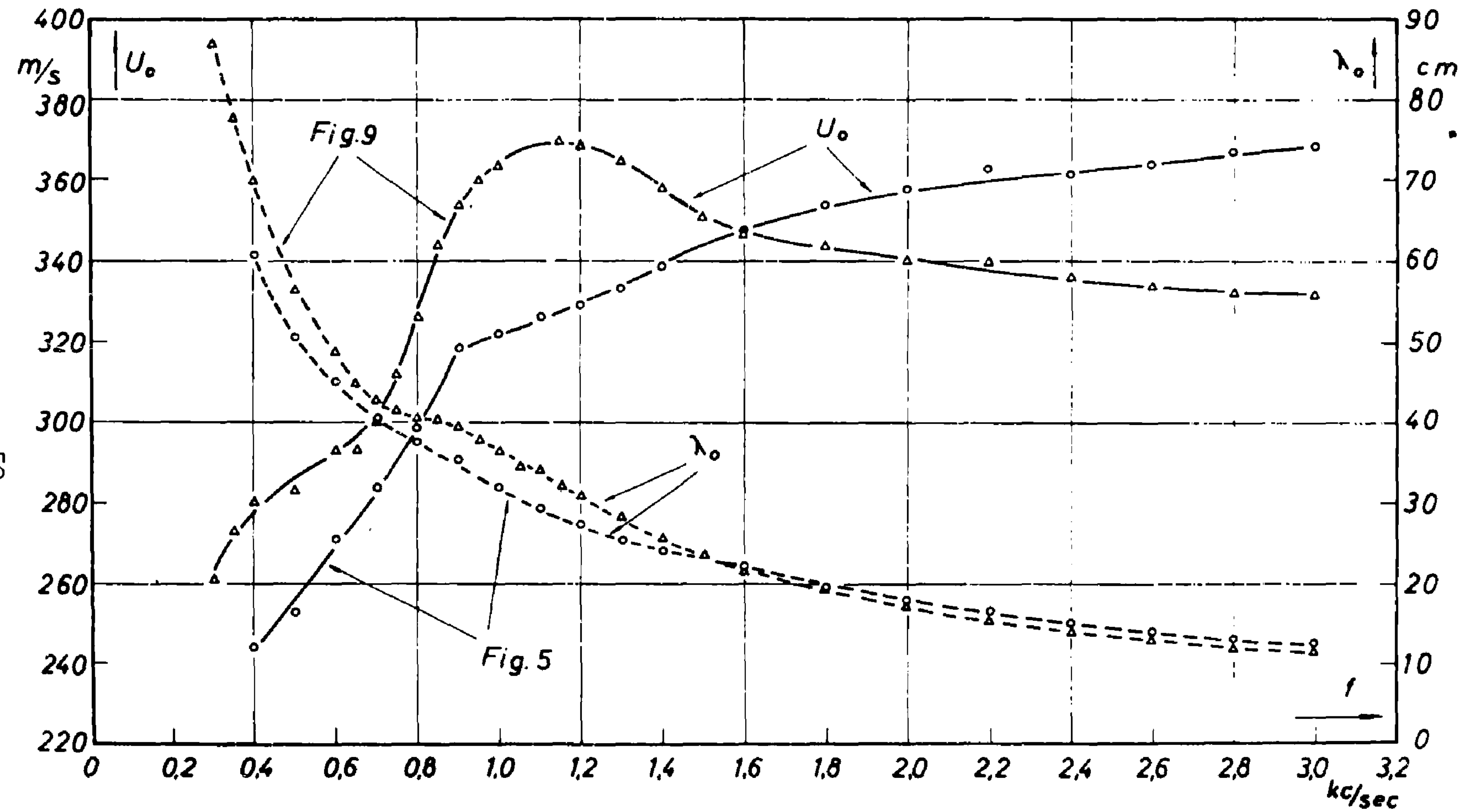


Fig.6: Phase velocity u_0 and wavelength λ_0 plotted against frequency f for a rock wool absorber accord. to Fig.5 and for an absorber consisting of damped Helmholtz resonators accord. to Fig.9.

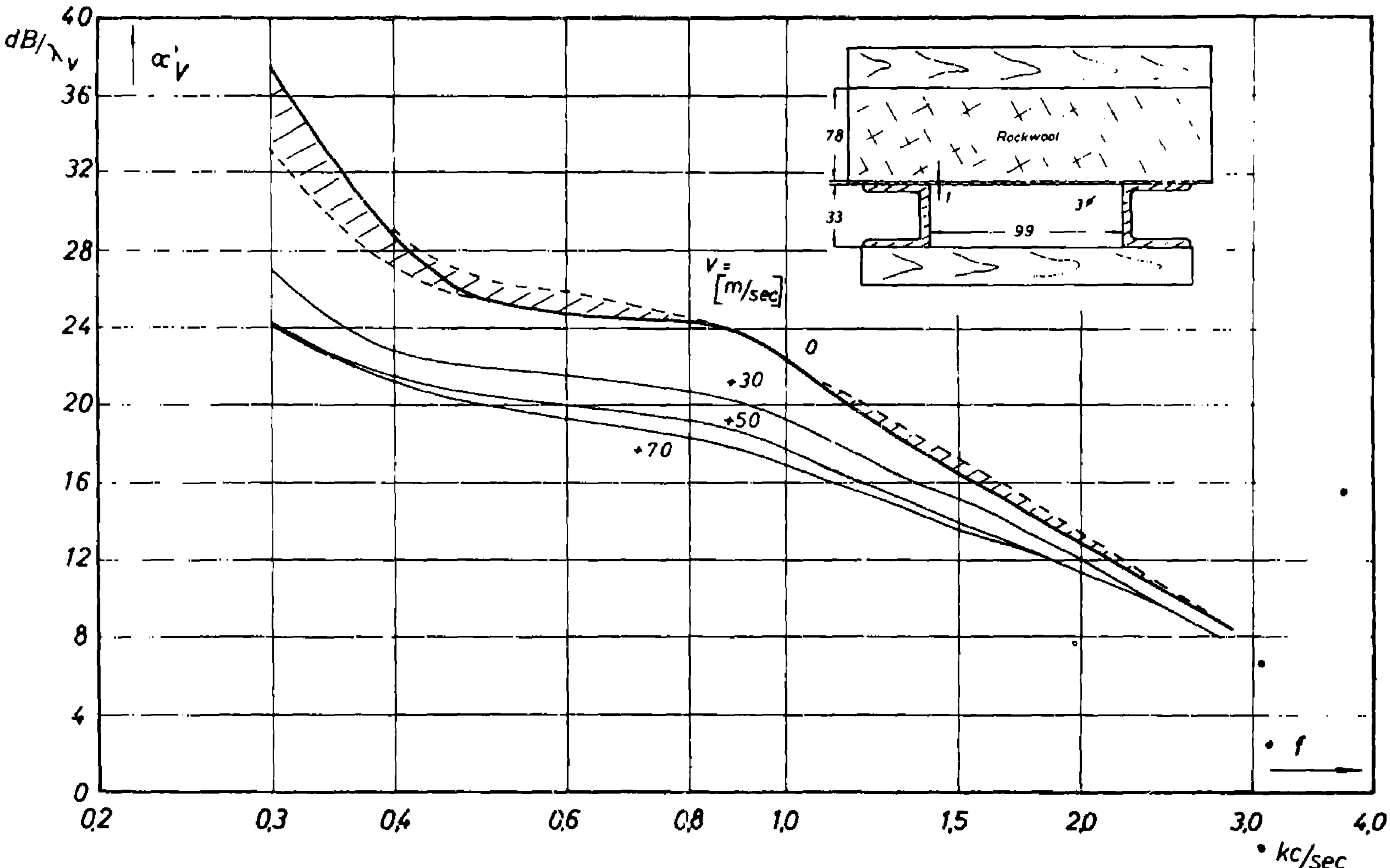


Fig.7: Attenuation α'_v per wavelength for flow velocities V versus frequency f of a rock woll absorber with perforated cover.

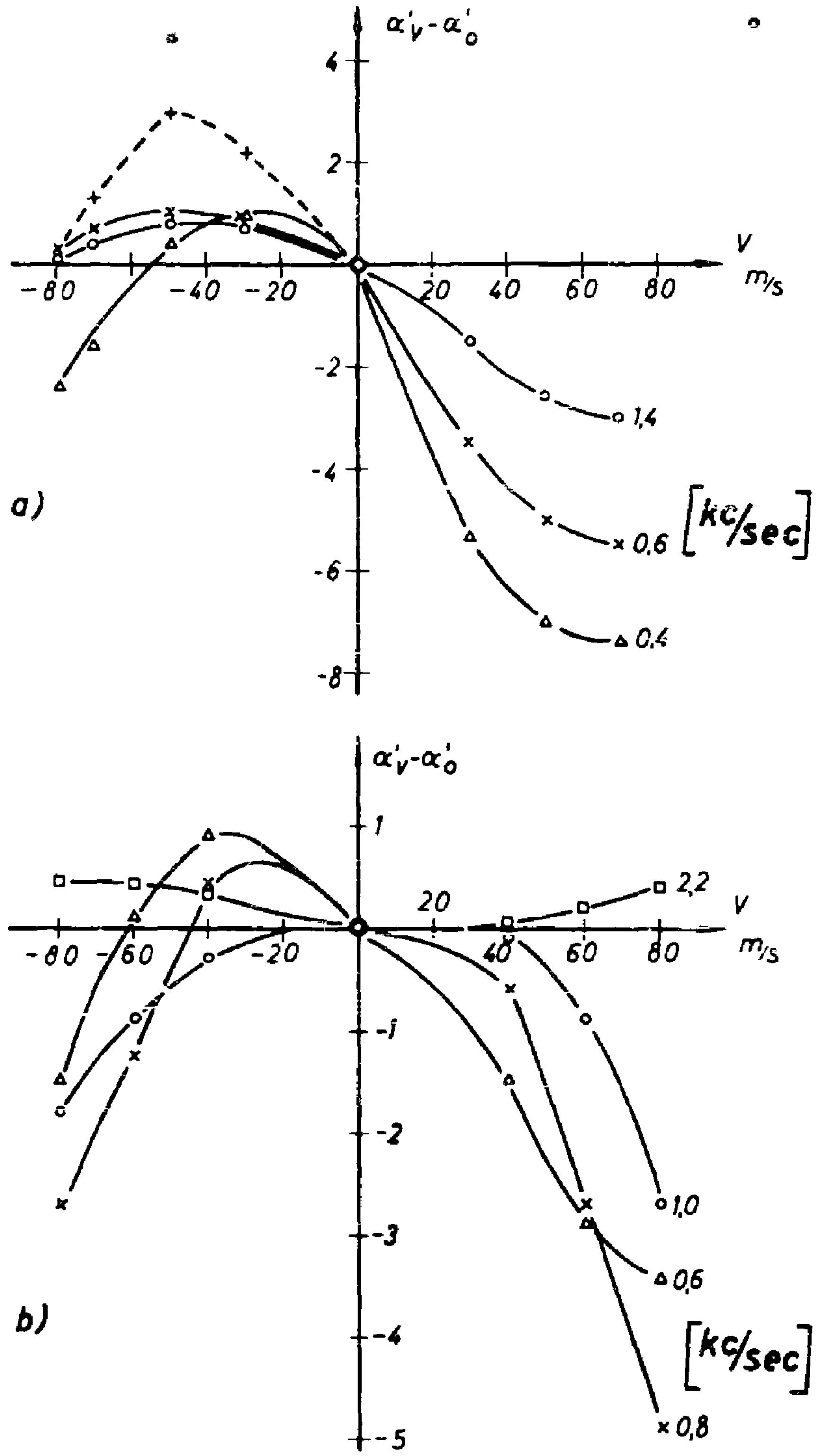


Fig.8: Change of the attenuation α'_V related to the wavelength as a function of the flow velocity V.

a) Rock wool absorber with perforated cover (Fig.5).

b) Absorber consisting of damped Helmholtz resonators (Fig.9.)

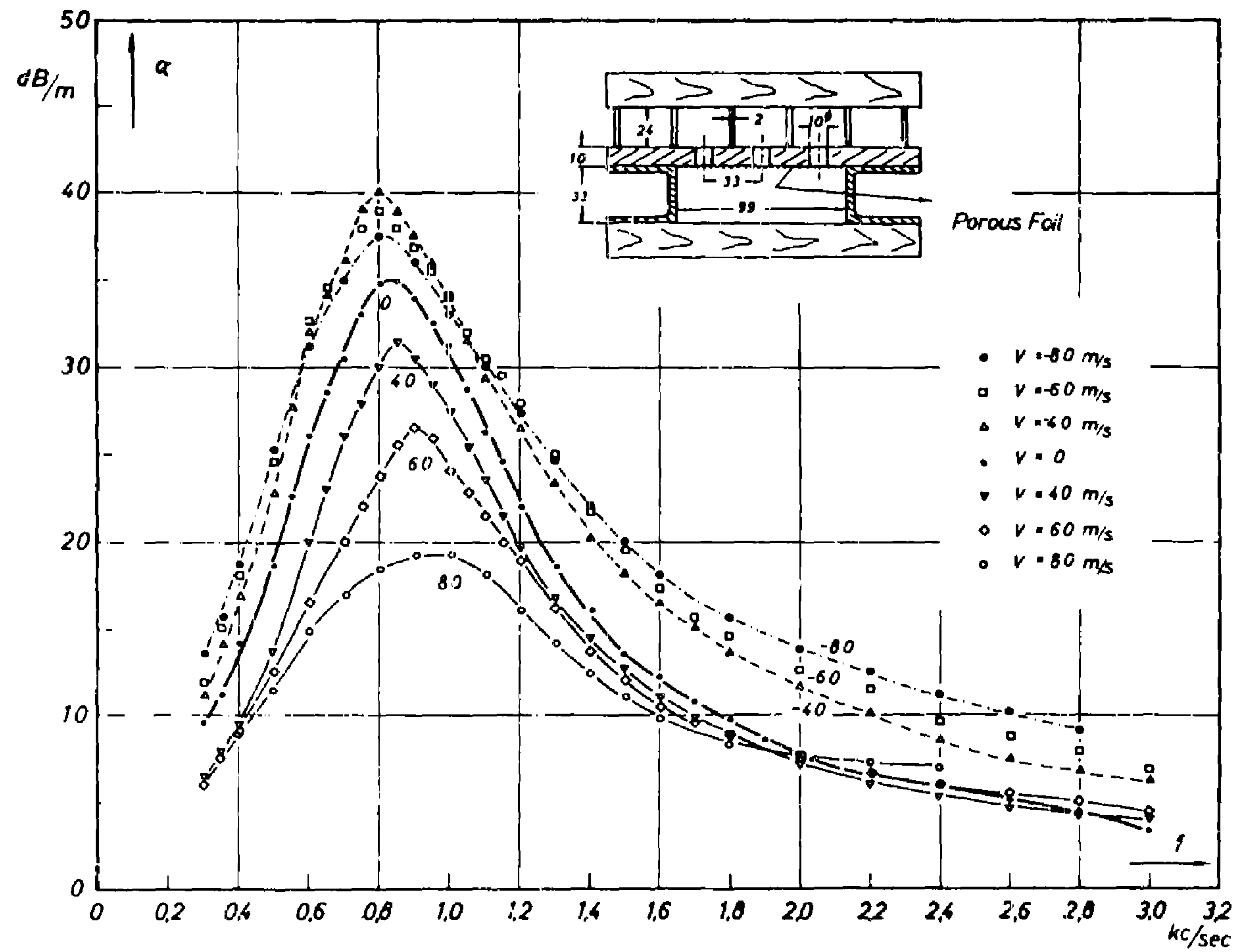


Fig.9.: Attenuation α versus frequency f of an absorber consisting of damped Helmholtz resonators (flow resistance in the necks by porous Mikropor-foil).

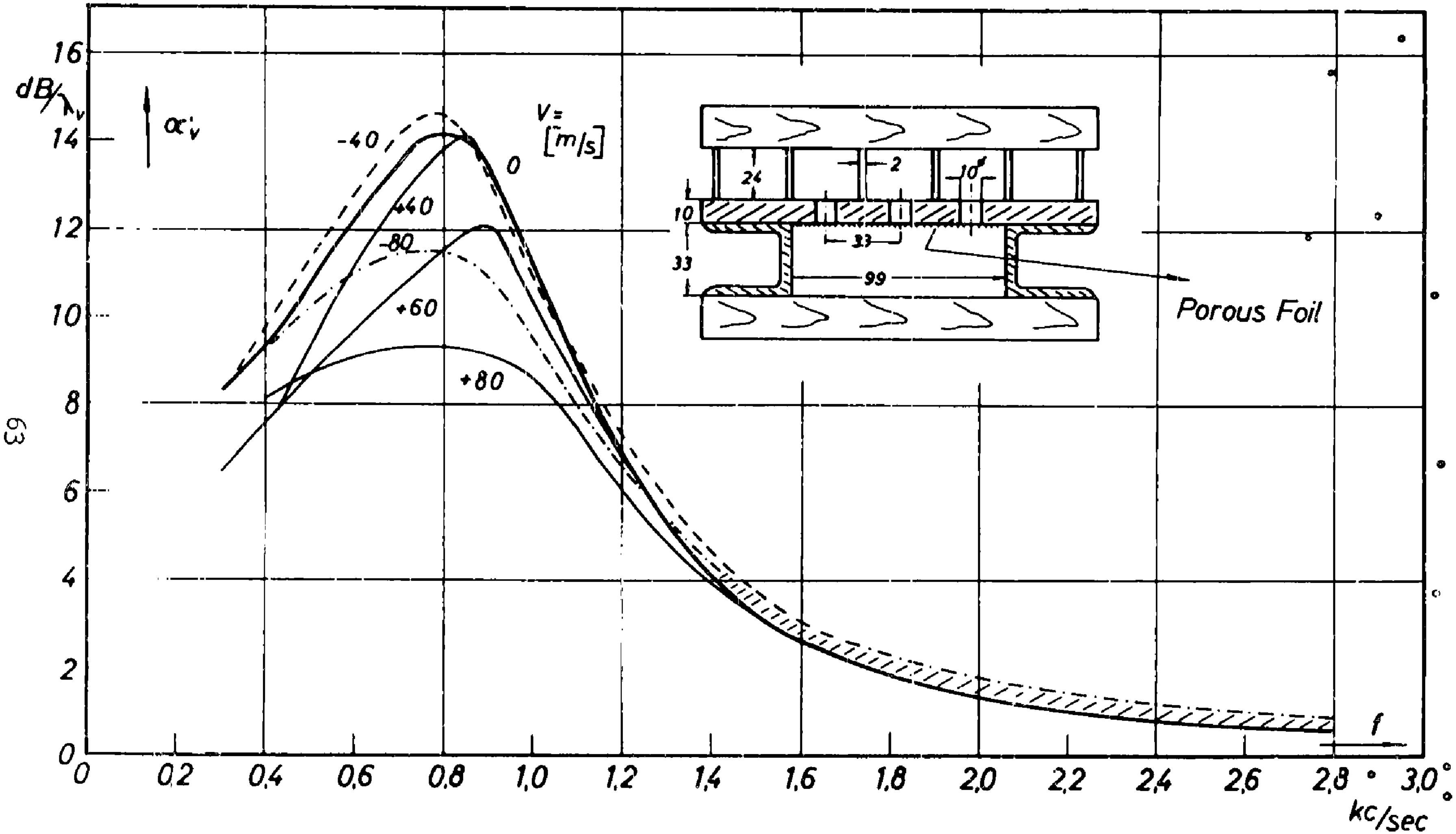


Fig.10: Attenuation α'_v per wavelength at flow velocities V versus frequency f of an absorber consisting of damped Helmholtz resonators.

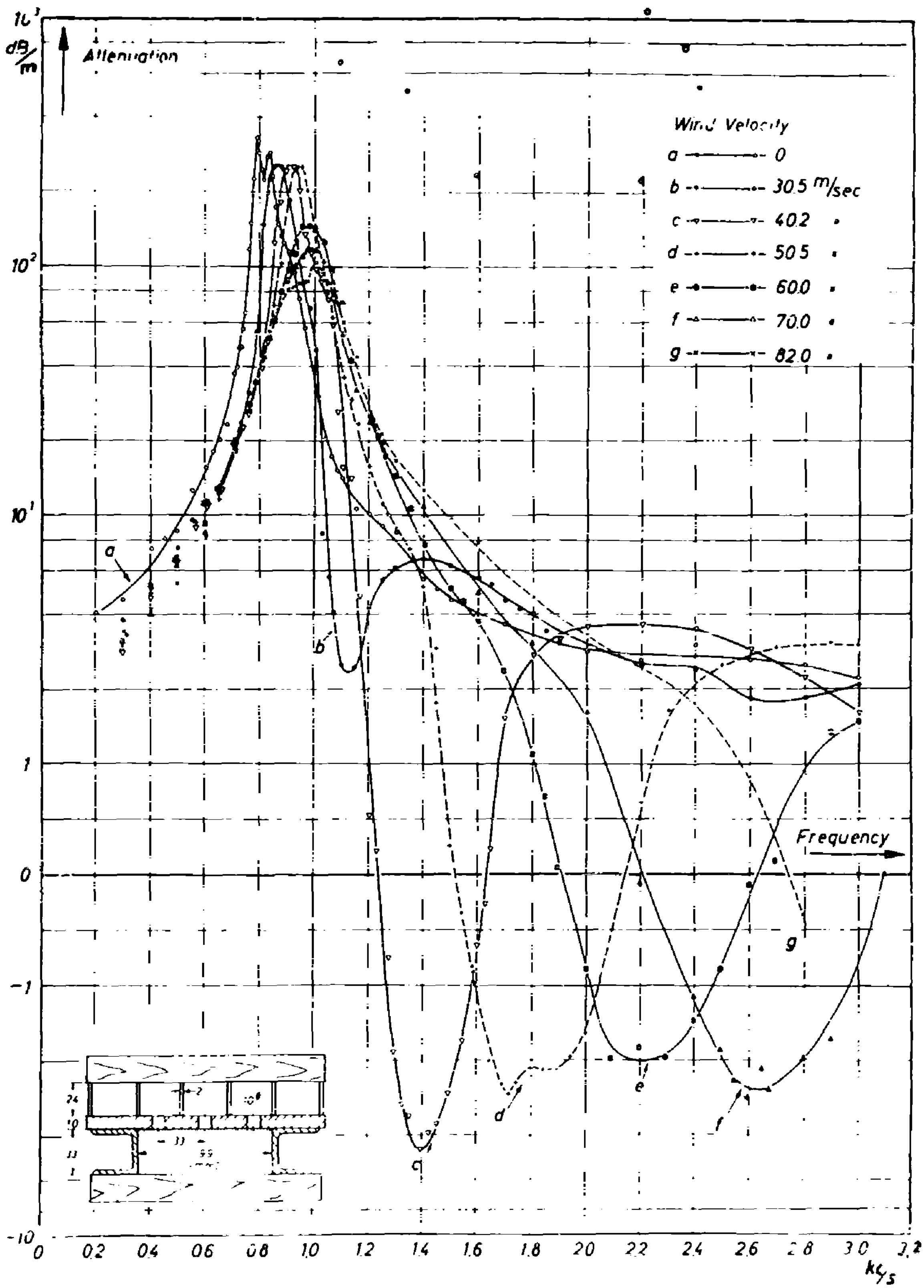


Fig. 11. Attenuation of Undamped Resonators versus Frequency
Parameter Wind Velocity $V (>0)$.

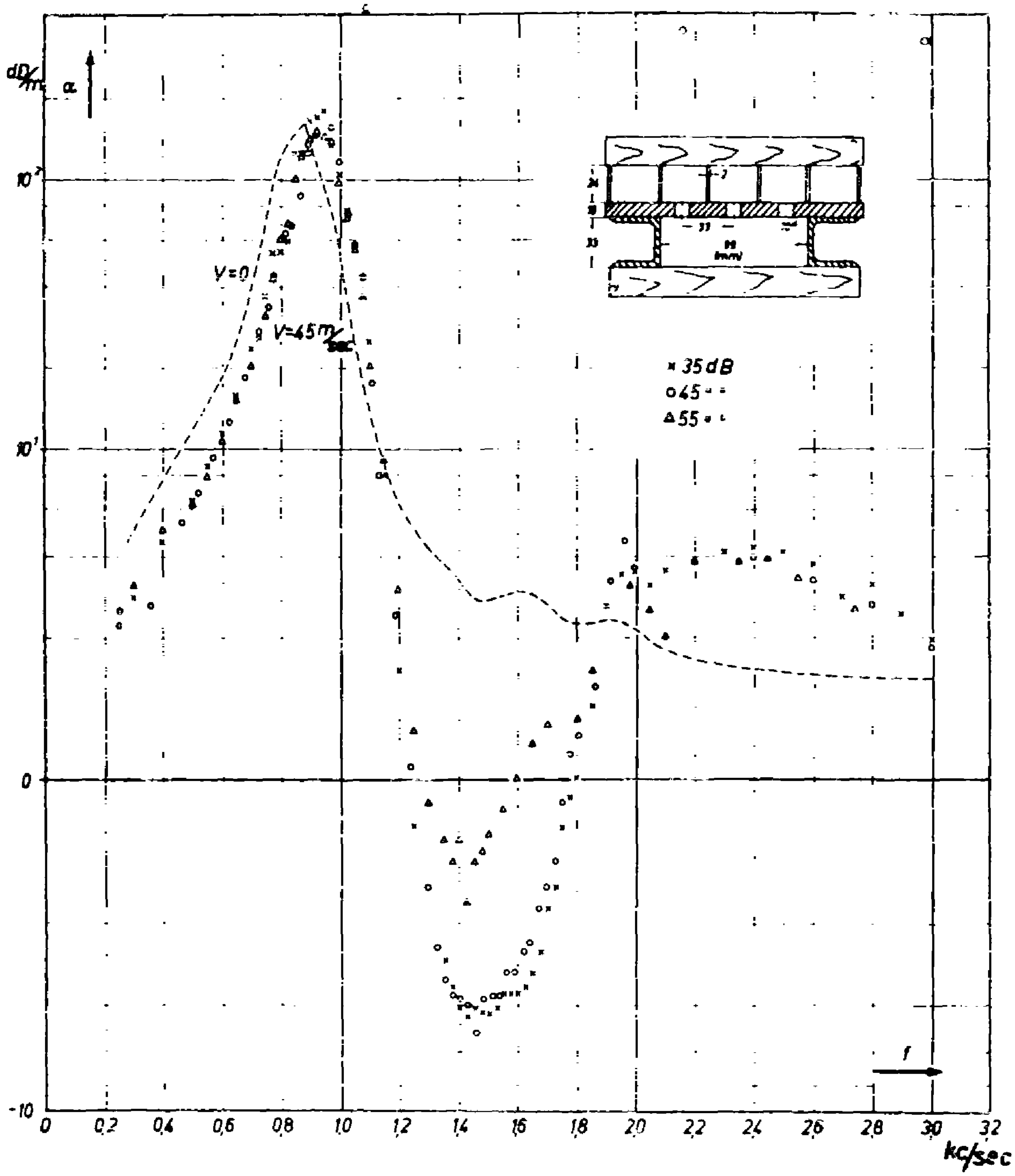


Fig.12: Measurements concerning the amplitude dependence of attenuation at flow velocity V= +45m sec and three signal levels. For comparison: attenuation in resting air. (V= 0, dotted line).

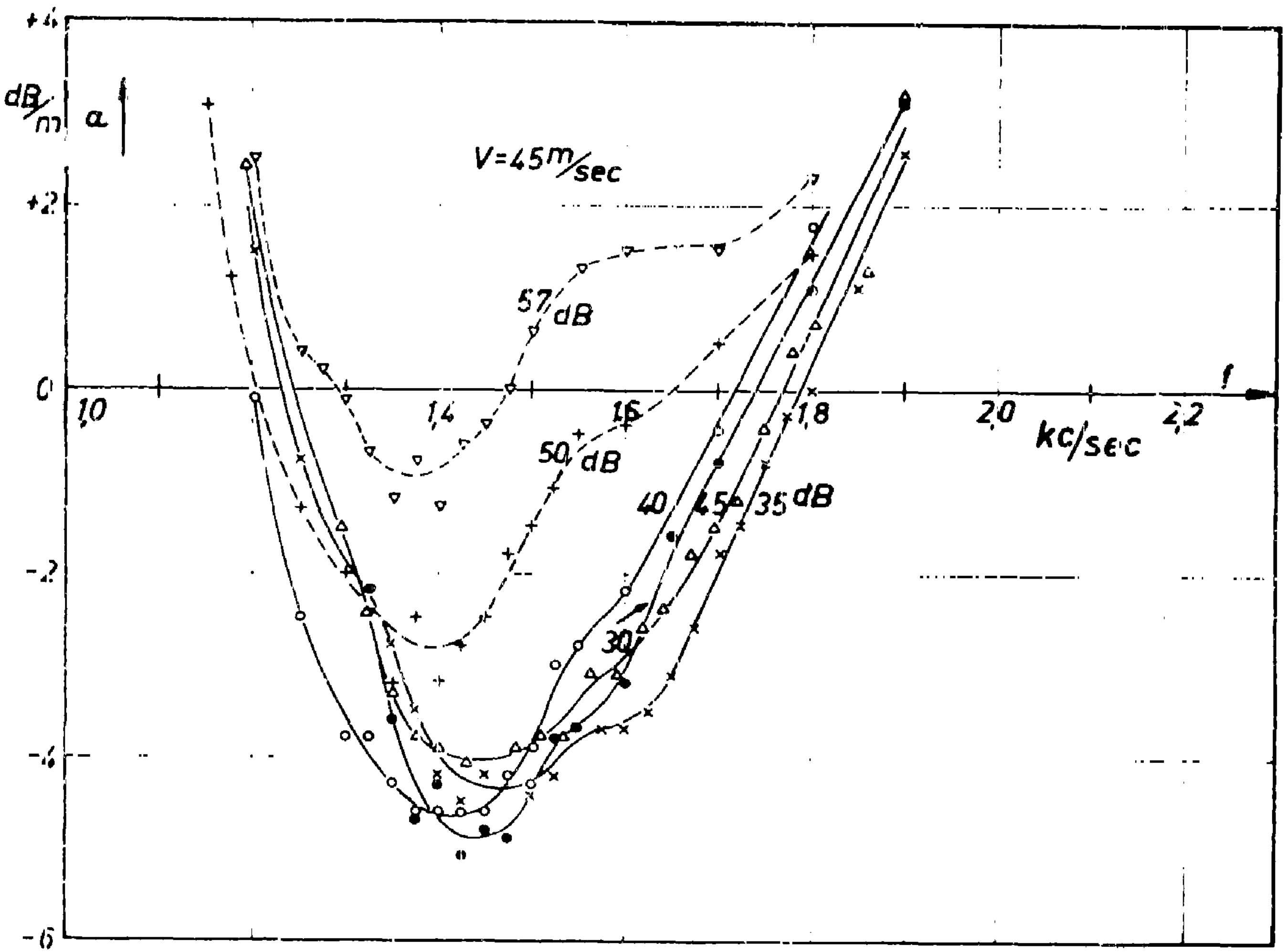


Fig.13: Amplitude dependence of sound amplification at flow velocity V=+45m/sec
Parameter: signal level.

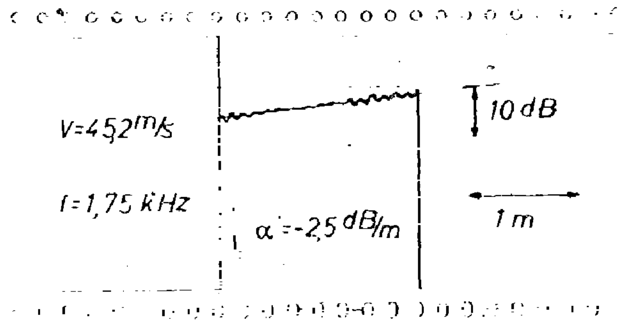


Fig.14.

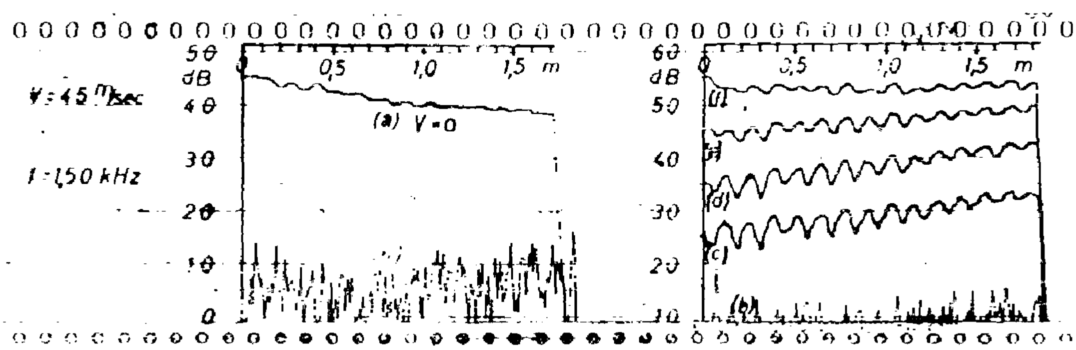


Fig.15

Records of the sound pressure when pulling the microphone through the duct coated with undamped Helmholtz resonators.

Fig.14: Flow velocity V=+45,2 m/sec; Frequency f =1,75 kc/sec

Amplification α = 25 dB/m.

Fig.15: Frequency f =15 kc/sec

(a) Signal at resting air.

(b) Flow noise.

(c)-(f) Signal at flow velocity V=+45 m/sec for different signal levels.

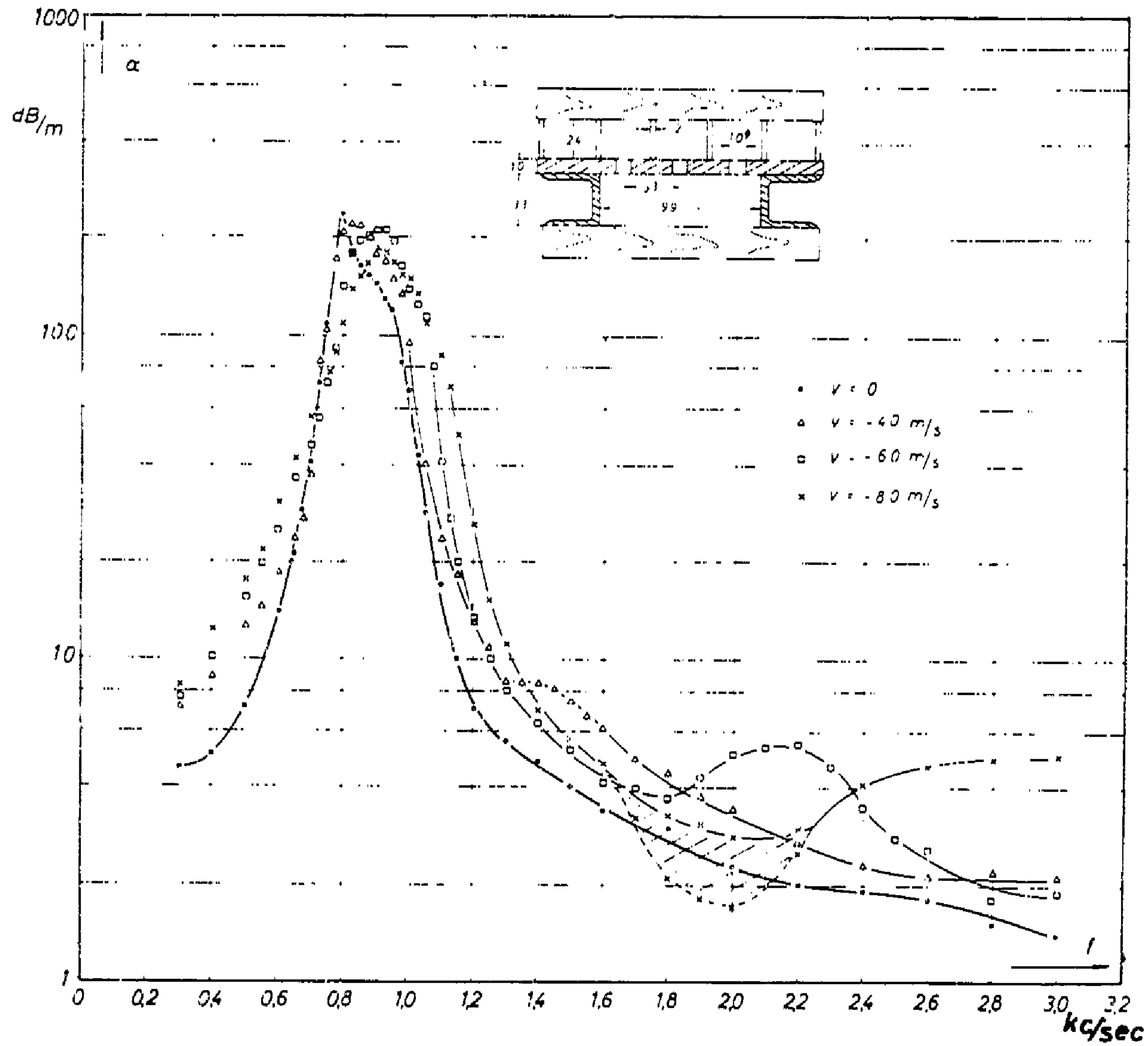


Fig.16: Attenuation α versus frequency f of undamped Helmholtz resonators for sound propagation against flow (V = 0). Hatched area: see text.

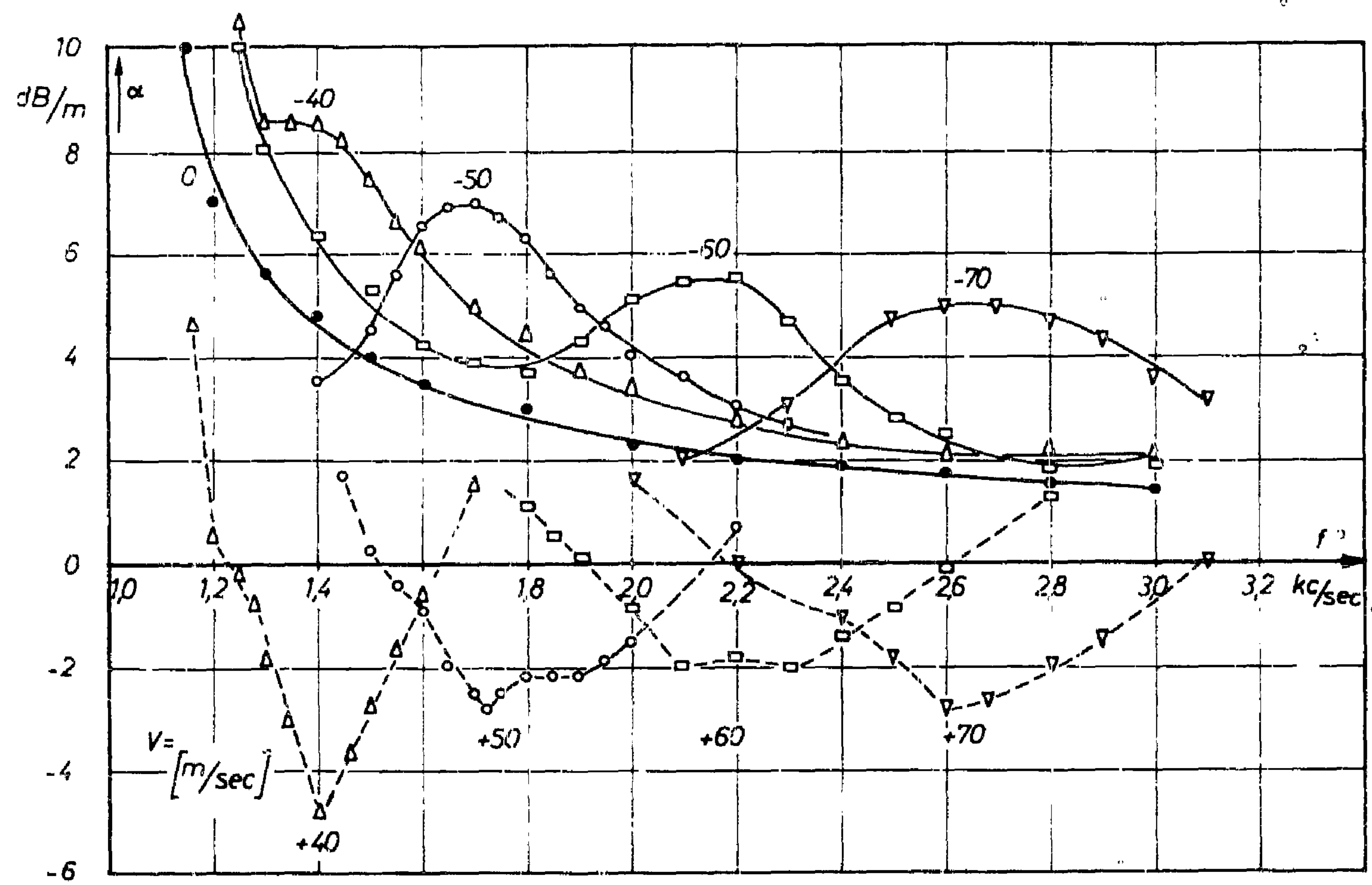


Fig.17: Attenuation and deattenuation maxima for sound propagation against flow direction (V<0) and in flow direction (V>0) of undamped Helmholtz resonators according to Fig.11.

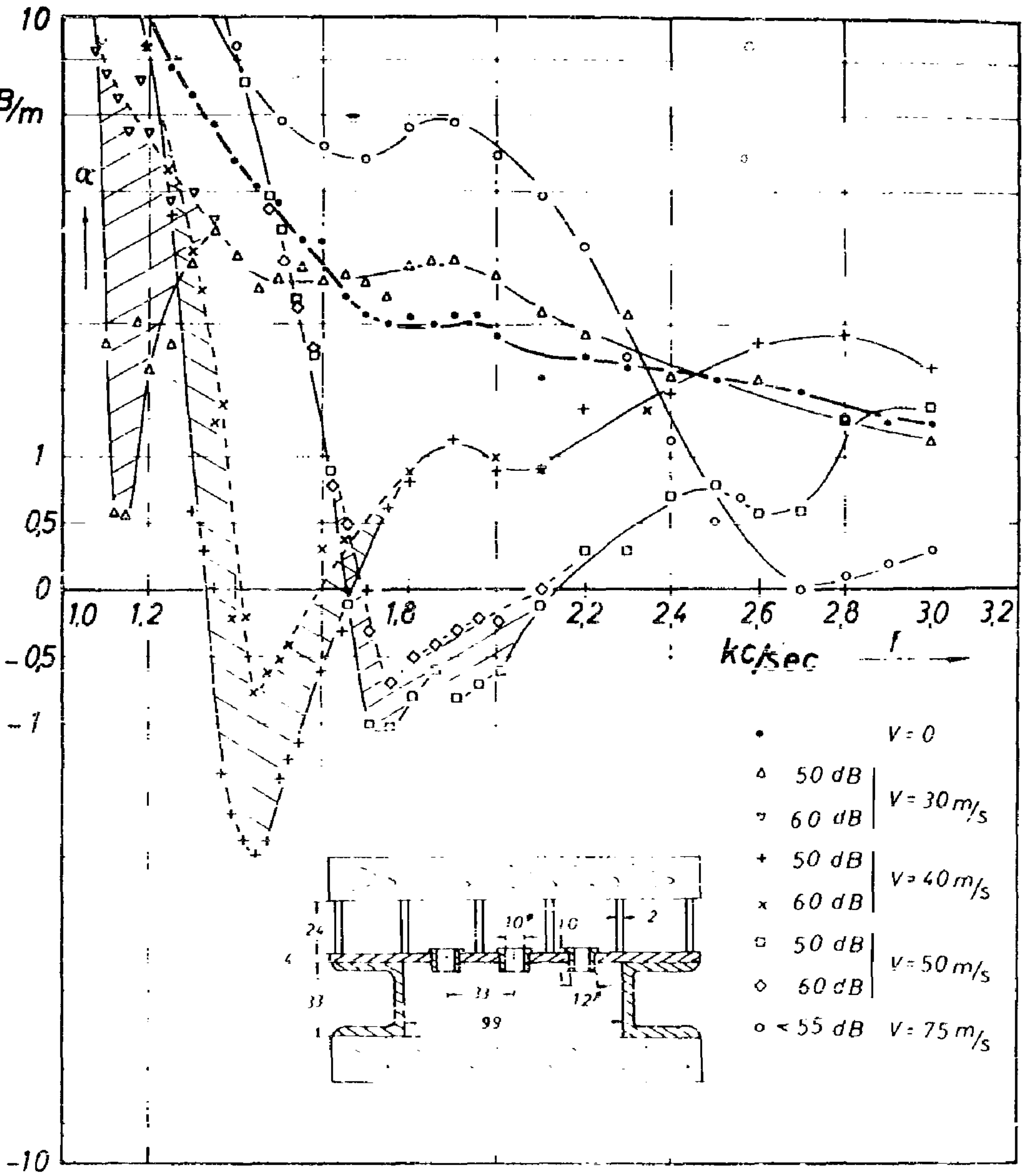


Fig.18: Attenuation α versus frequency f for an absorber consisting of Helmholtz resonators with protruding necks. Parameter: flow velocity V (≥ 0) and signal level.

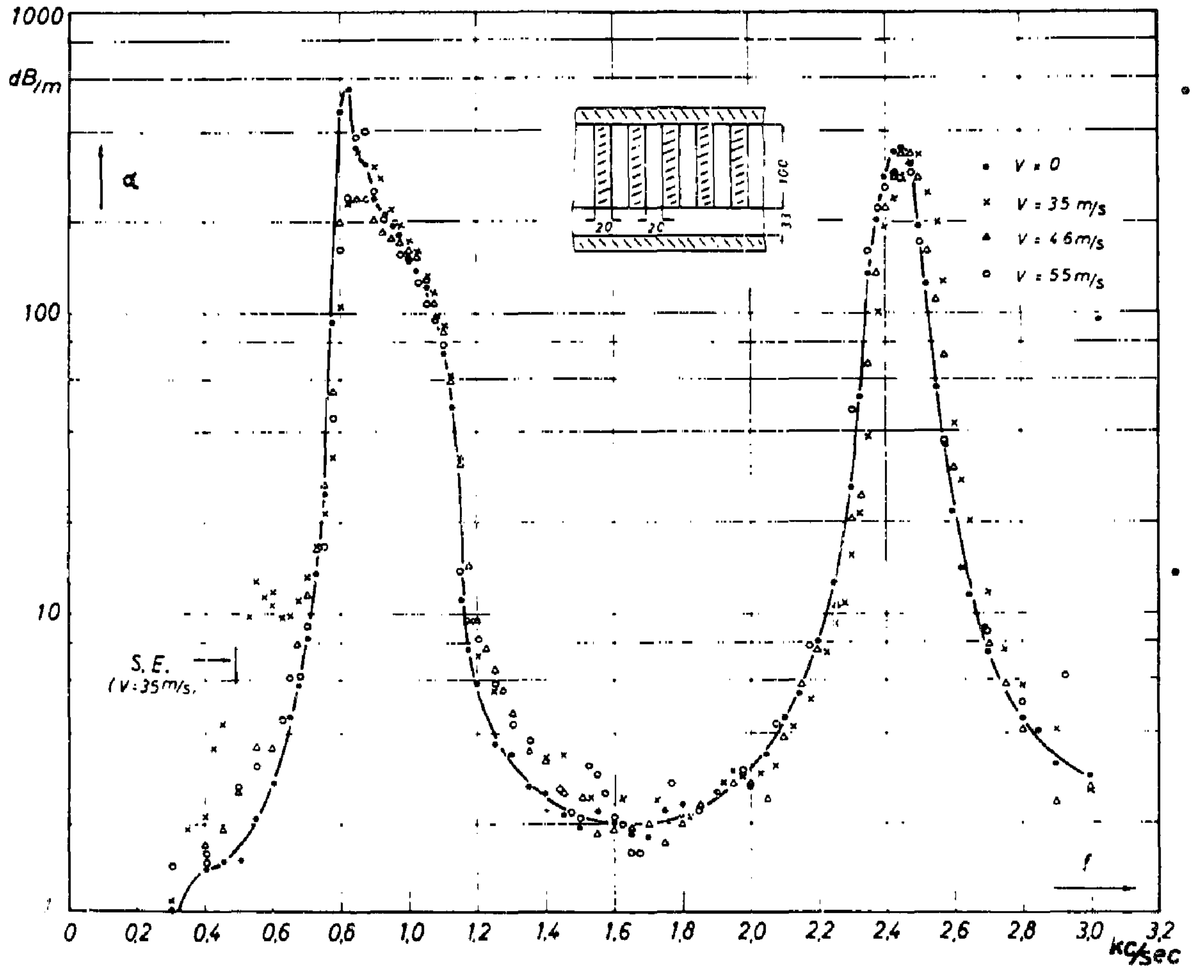


Fig.19: Attenuation α versus frequency f of a single sided „comb line"
Parameter: flow velocity V (≧ 0).
S.E.: Self-excitation frequency of duct for flow velocity V=+35 m/sec

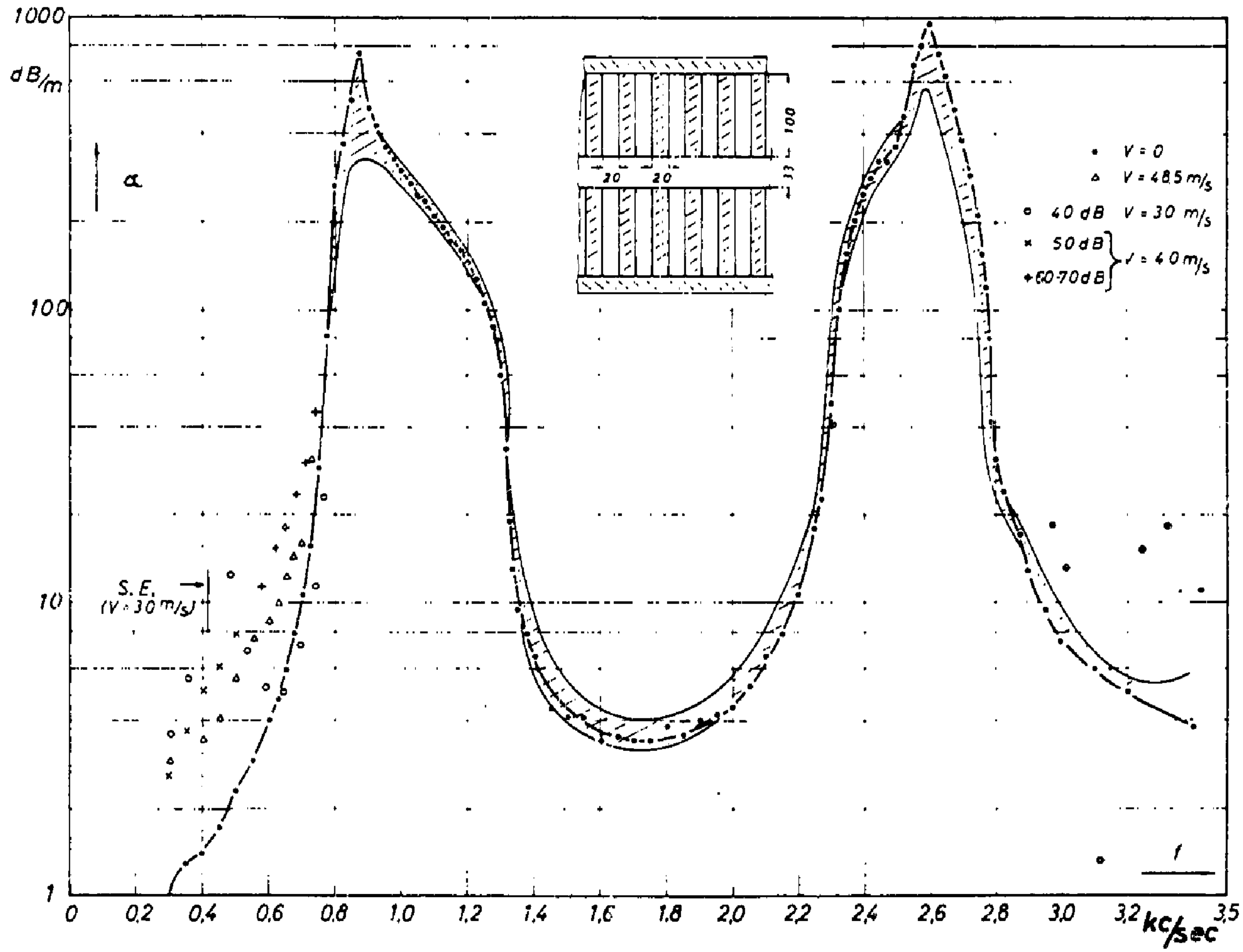


Fig. 20: Attenuation α versus frequency f of a double-sided „comb line". Parameter: flow velocity V (≧ 0) and signal level.

The hatched area contains all measuring points at the given flow velocities V.

S.E.: Self-excitation frequency of duct for flow velocity V = +30 m/sec.

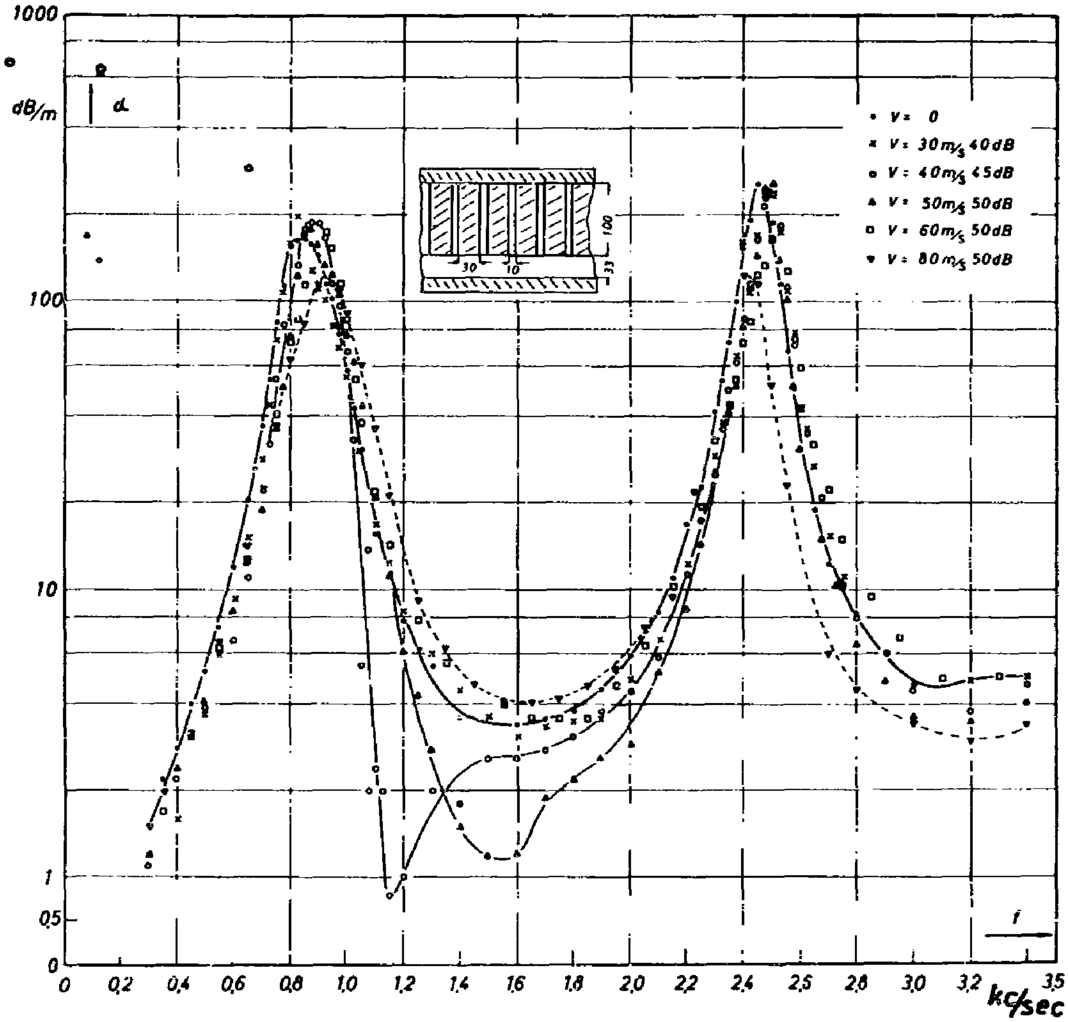


Fig.21: Attenuation α versus frequency f of a single-sided „comb line" with smaller slits. Parameter: flow velocity V (≧ 0) and signal level.

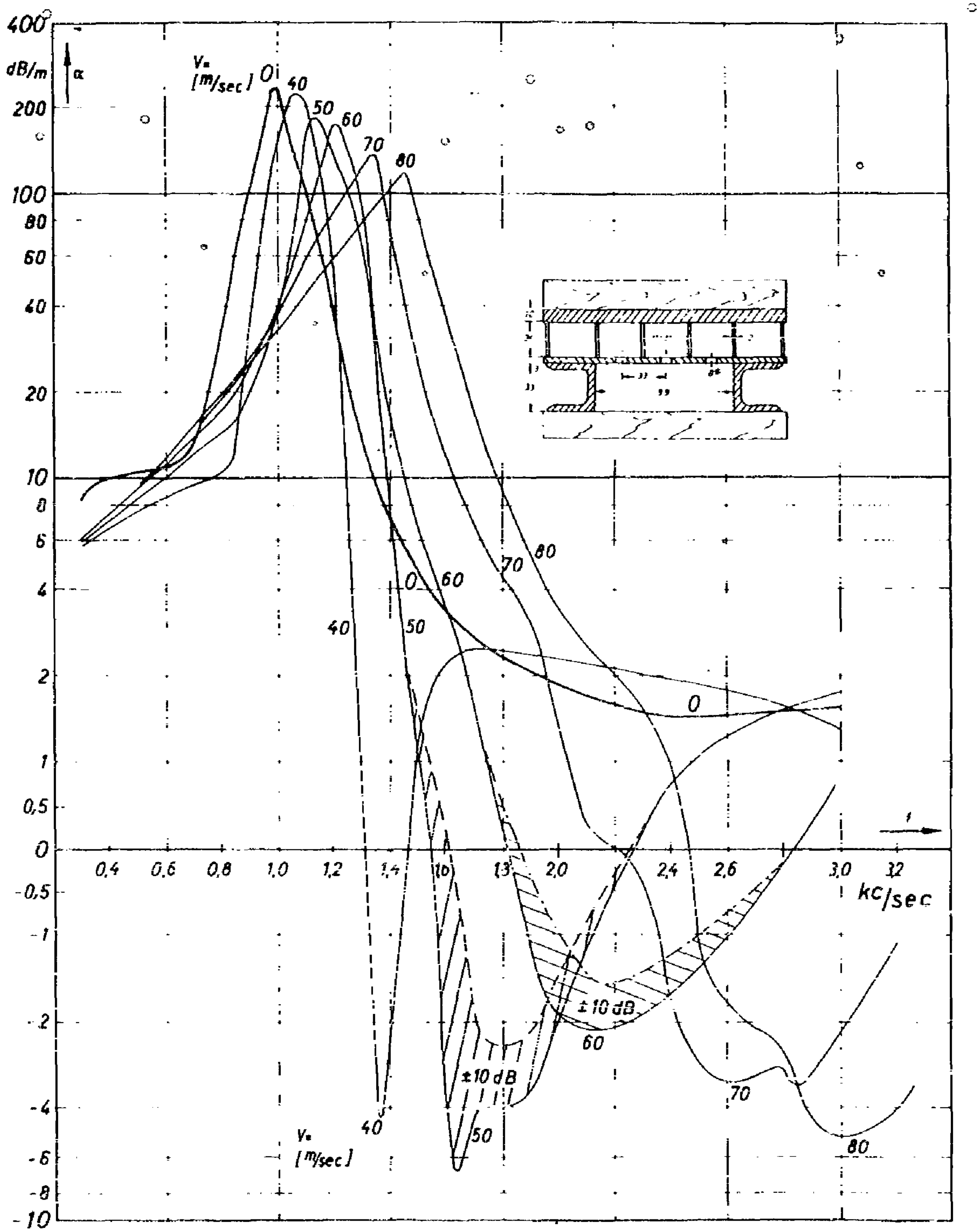


Fig.22: Attenuation α versus frequency f of an absorber consisting of Helmholtz resonators with short necks. Parameter: flow velocity V (≧ 0). Hatched area: Change of α caused by change of signal level of ± 10 dB.

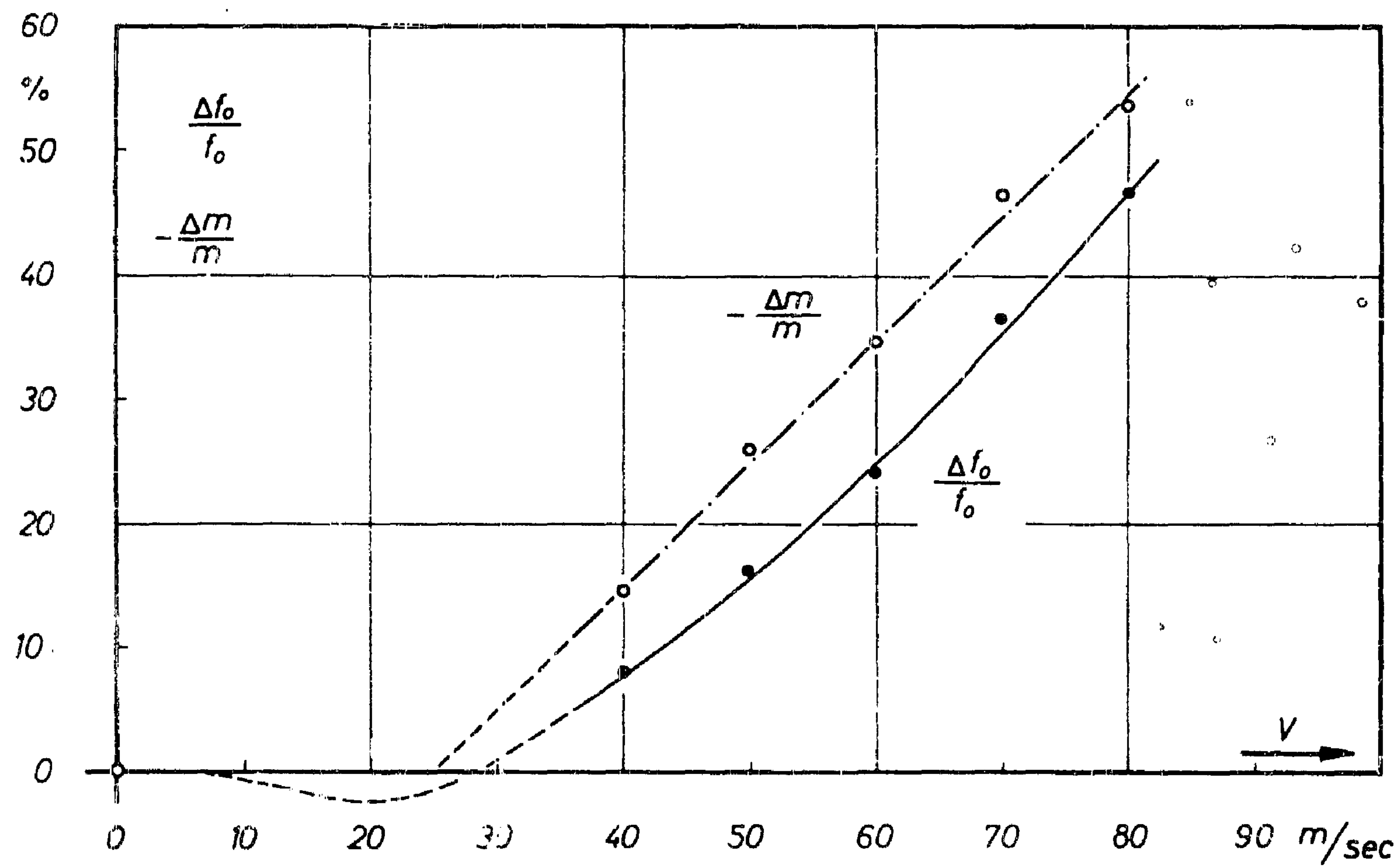


Fig.23: Relative change of resonance frequency f_o and the derived vibrating mass m of a single Helmholtz resonator with short neck according to Fig.22 caused by flow with the flow veloity V.

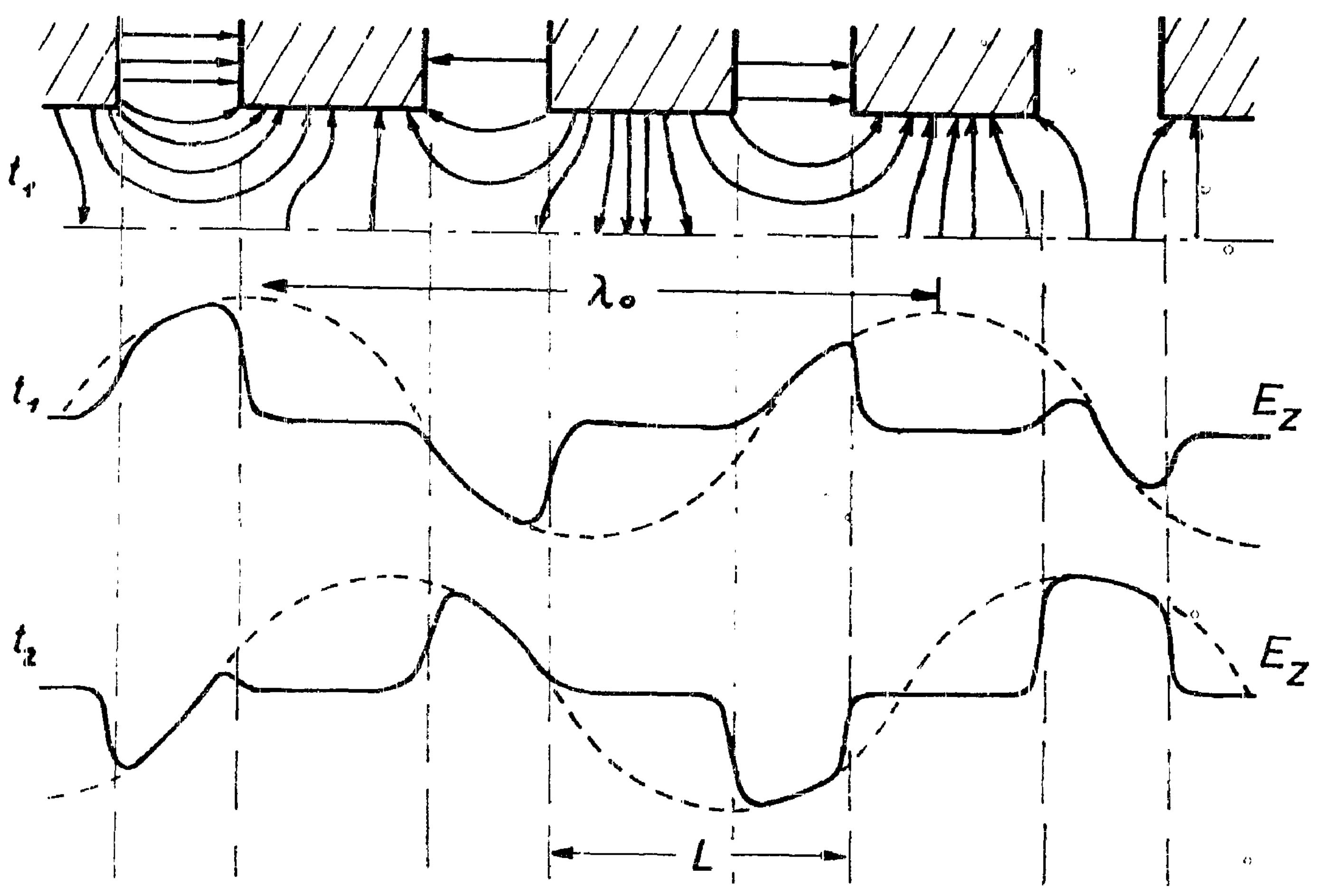


Fig. 24: Field pattern of electrical field strength in front of a „comb line".

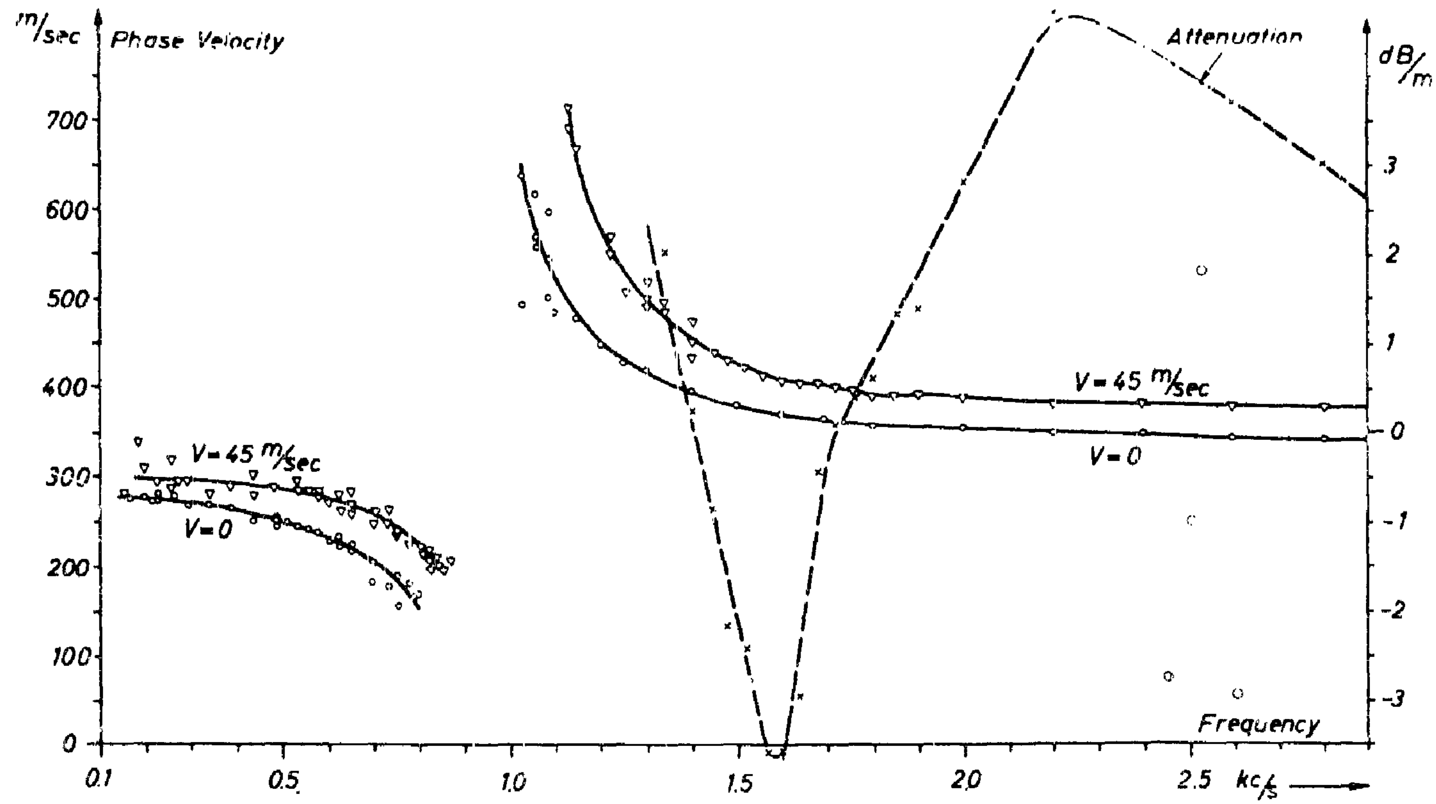


Fig. 25: Phase velocity of the fundamental wave in the duct with undamped Helmholtz resonators accord. to Fig.11 in resting (V= 0) and in streaming air (V=+45 m/sec) and attenuation measured simultaneously as a function of frequency f.

Thin dotted line: dispersion curve for streaming air shifted downwards by the value of the flow velocity V=+45 m/sec.

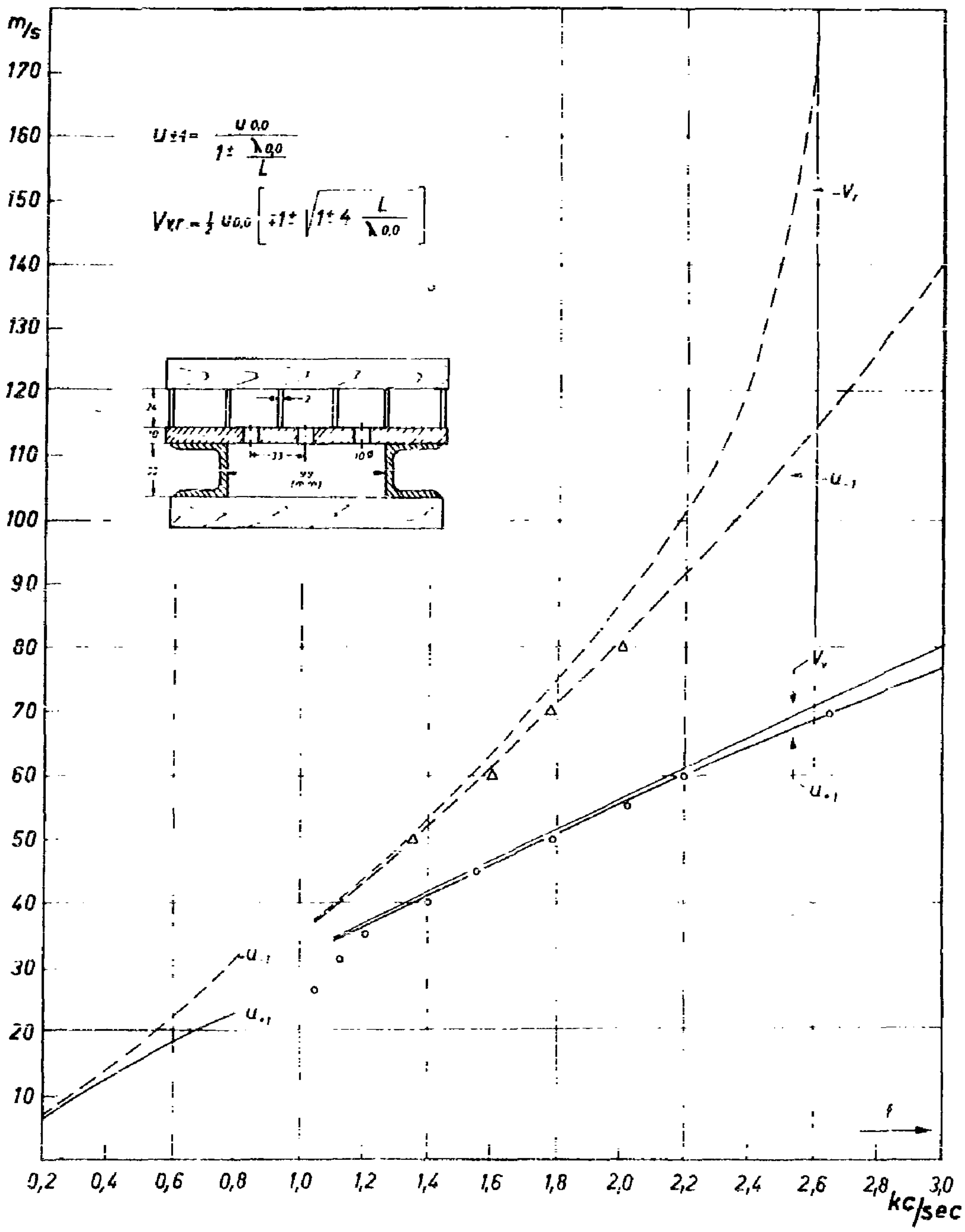


Fig. 26: Comparison of points of maximal deattenuation with the dispersion curve of the first partial waves. Duct with undamped Helmholtz resonators.

○ frequency and flow velocity for maximal amplification with forward propagation ($V > 0$).

△ frequency and absolute amount of flow velocity for maximal deattenuation with backward propagation ($V < 0$).

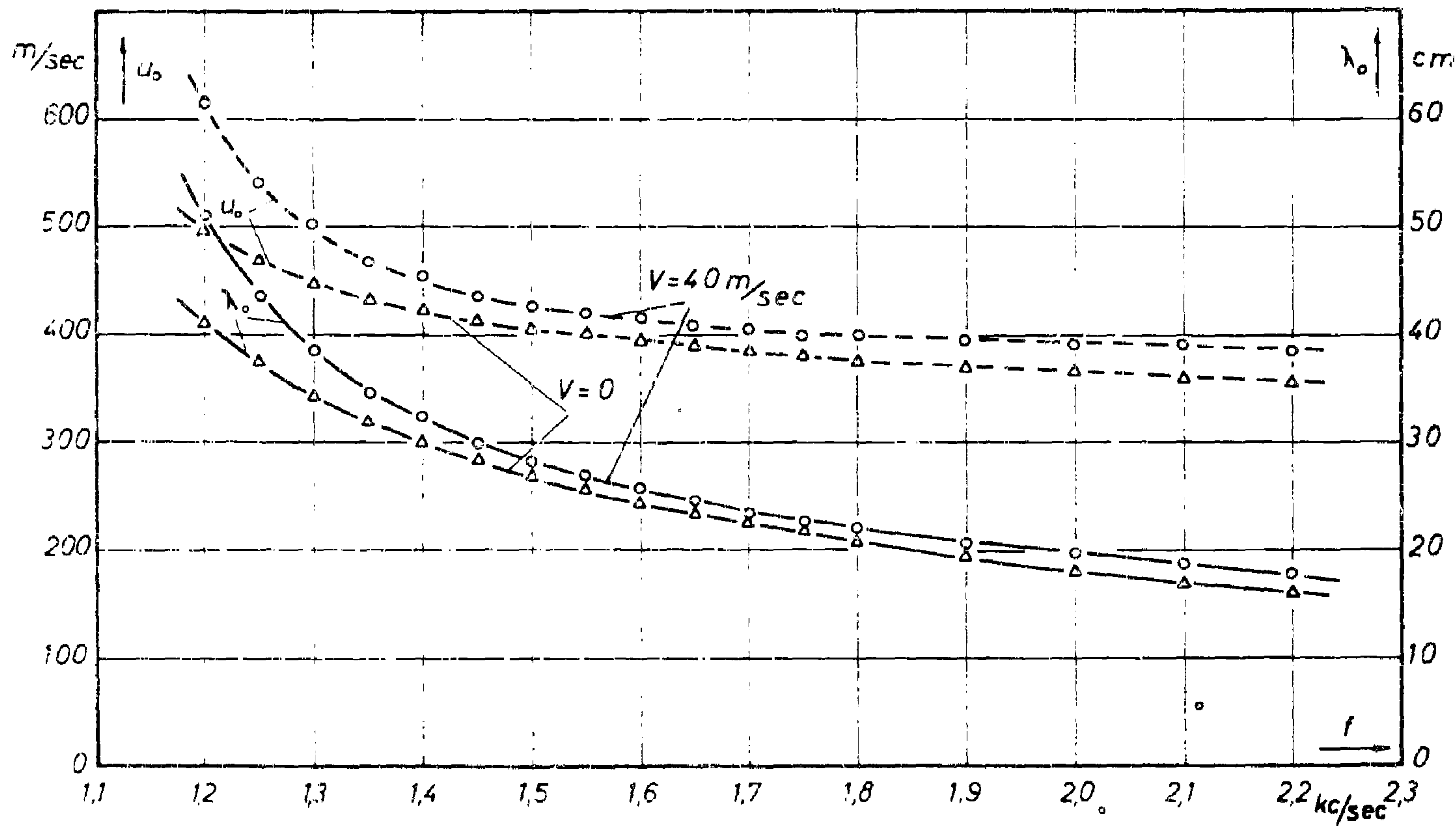


Fig.27: Phase velocity u_o and wavelength λ_o of the fundamental wave in the duct coated with Helmholtz resonators with protruding necks accord. to Fig.18. Parameter: flow velocity V.

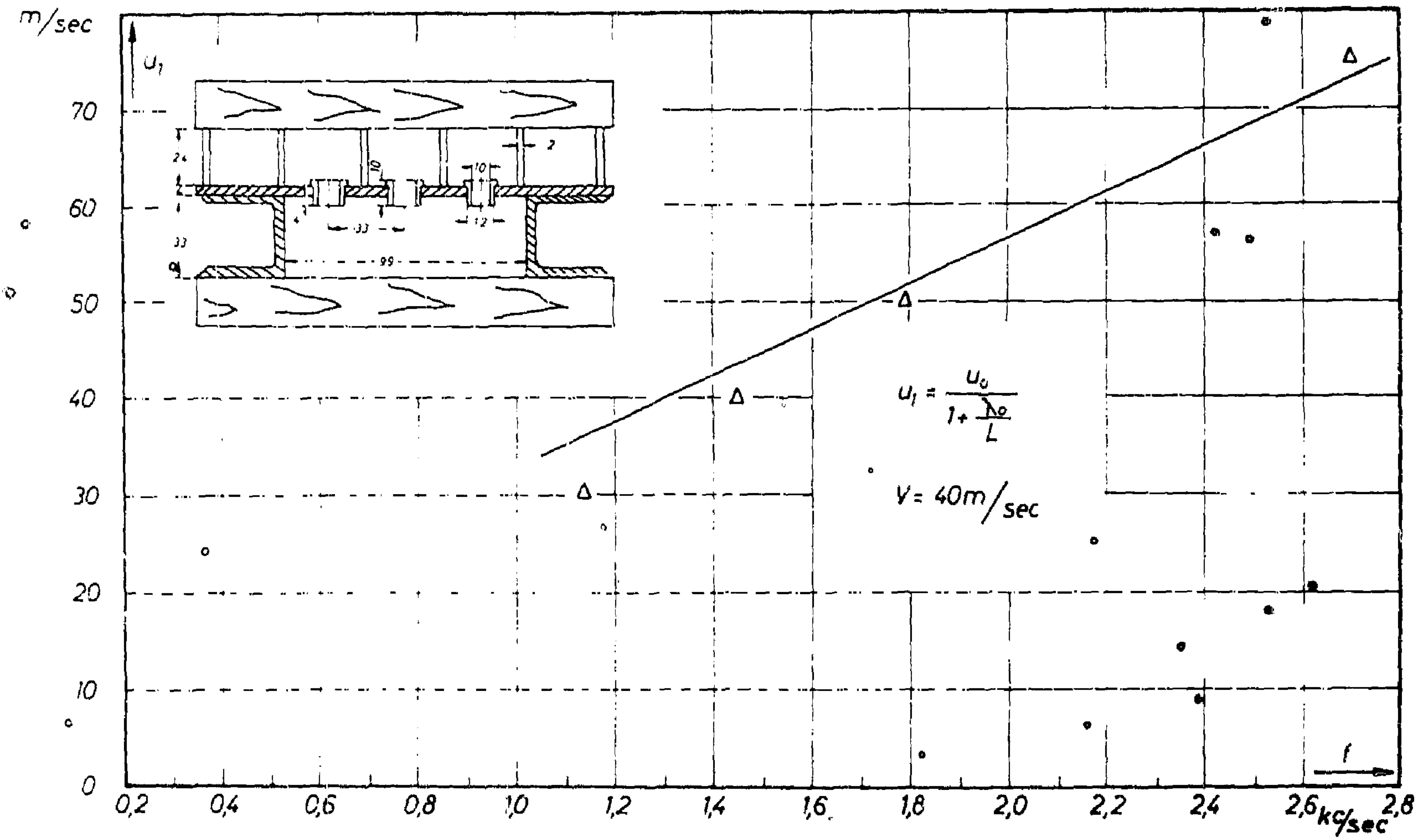


Fig. 28: Comparison of the points of maximal deattenuation with the phase velocity u_1 of the first partial wave with resonators with protruding necks.

Curve: u_1 calculated with u_0 and λ_0 at $V = +45$ m/sec.

Δ flow velocity V and frequency f for maximal deattenuation.

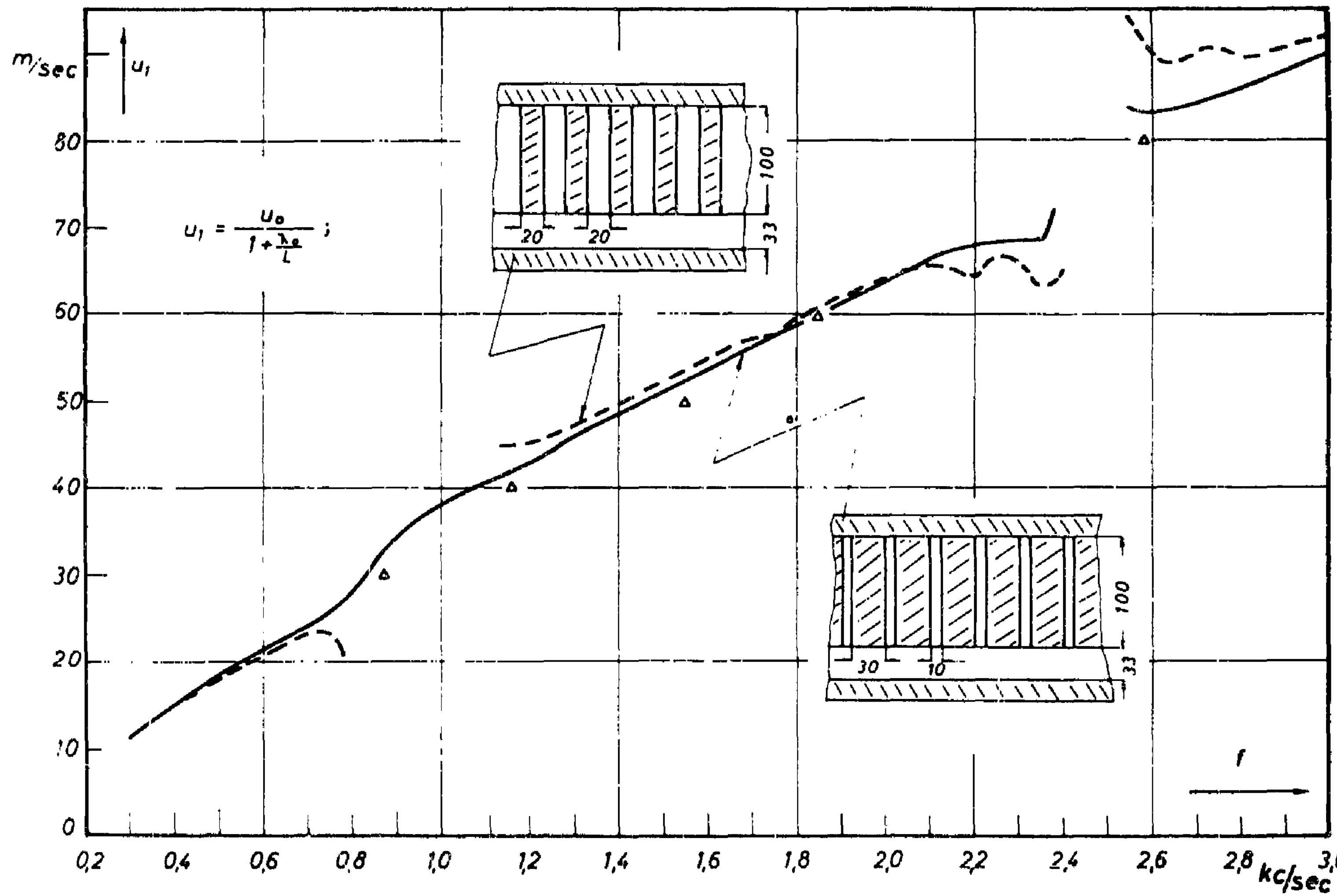


Fig.29: Phase velocity u_1 of the first partial wave in ducts with single-sided „comb line" △ flow velocity V and frequency f for maximal deattenuation in the „comb line" with narrow slits.

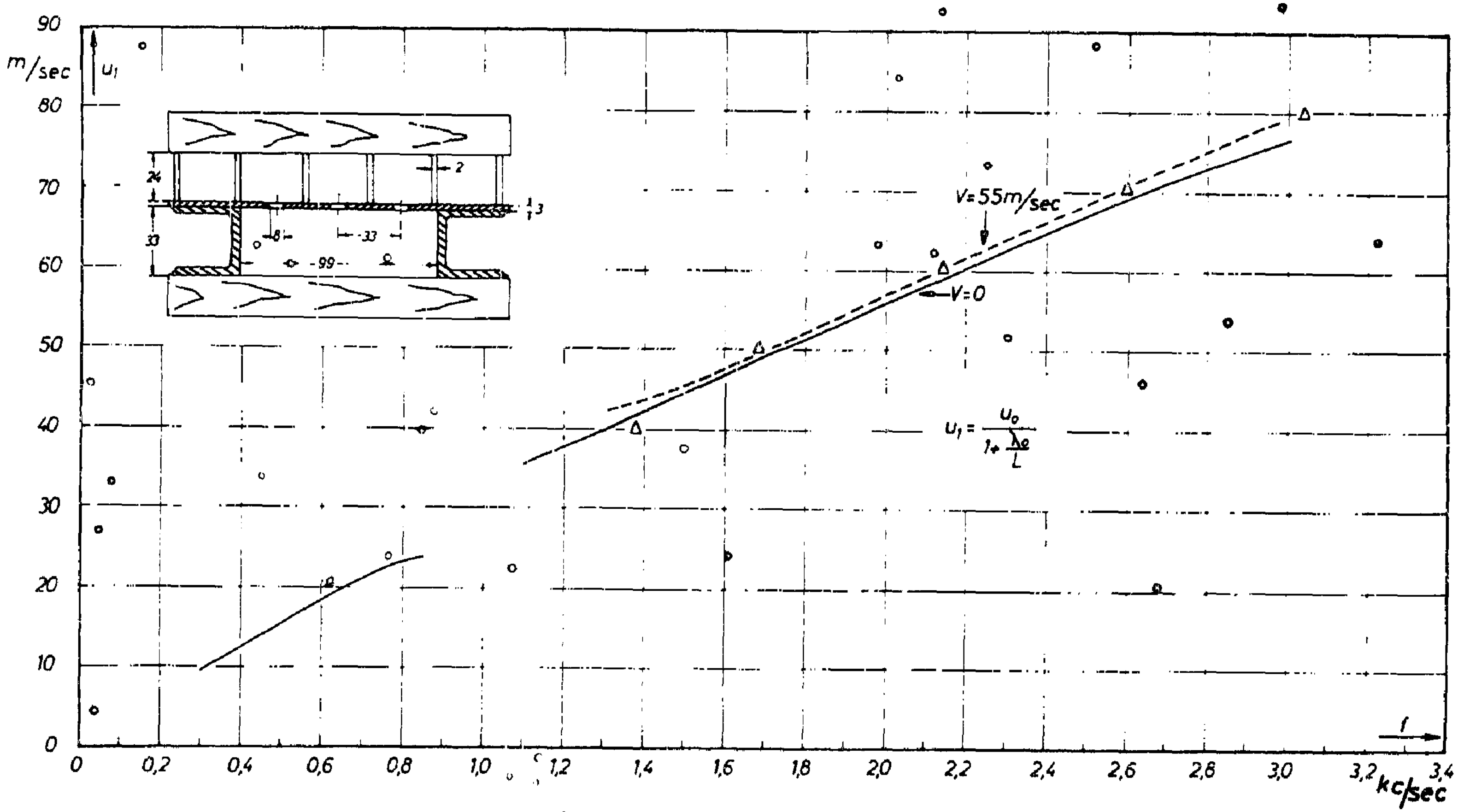


Fig.30: Phase velocity u_1 of the first partial wave in the duct with Helmholtz resonators with short necks in resting (V=0) and in streaming air (V=+55 m/sec).
Δ flow velocity V and frequency f for maximal deattenuation.

Appendix I.

Differential Equation for the Alternating Component of the Mass Flow Density in a Duct with Variable Boundary.

The flow $V(z,t)$ is seperated into a stationary fundamental flow V_o and an alternating term $V_1(z,t)$ which includes particle velocity as well as an eventual flow pulsation. The coordinate dependence of the duct boundary is taken into account with respect to its influence on the fundamental flow by setting up the latter as a function of z. The problem is limited to a uni-dimensional flow in z-direction. The conditions enumerated in the text be imposed. All values are a sum of the stationary term (index o) and the alternating term (index 1). The alternating term is proportional to $e^{j\omega t}$.

As fundamental equation we use the equation of force:

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + (V\,\nabla)V = -\frac{1}{\rho}\cdot\nabla p \qquad (1A)$$

the equation of continuity

$$\operatorname{div} J = -\frac{\partial\rho}{\partial t} \qquad (J = \text{mass flow density}) \quad (2A)$$

the adiabatic equation of state in the form

$$p_1 = c^2\cdot\rho_1 \qquad (p = \text{pressure}) \qquad (3A)$$

and the defining equation for the mass flow density

$$J_o = \rho_o V_o \;;\quad J_1 = \rho_1 V_o + \rho_o V_1 \qquad (4A)$$

For the uni-dimensional case with the presupposed time dependence follows after separation from the equation valid for the stationary values

$$j\omega V_1 + V_o\frac{\partial V_1}{\partial z} + V_1\frac{dV_o}{dz} = -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_1 \qquad (5A)$$

$[\,..\,]_1$ = alternating terms of first order.

$$\frac{\partial J_1}{\partial z} = - j\omega \varrho_1 \qquad (6A)$$

From (4A) and (6A) follows

$$V_1 = \frac{J_1}{\varrho_o} - \frac{\varrho_1}{\varrho_o} V_o = \frac{V_o}{J_o} J_1 - \frac{V^2_o}{J_o} \varrho_1 = \frac{V_o}{J_o} J_1 + \frac{1}{j\omega} \frac{V^2_o}{J_o} \frac{\partial J_1}{\partial z} \qquad (7A)$$

whence with $\partial J_o / \partial z = 0$

$$\frac{\partial V_1}{\partial z} = \frac{1}{j\omega} \frac{V_o^2}{J_o} \frac{\partial^2 J_1}{\partial z^2} + \frac{2}{j\omega} \frac{V_o}{J_o} \frac{dV_o}{dz} \frac{\partial J_1}{\partial z} + \frac{V_o}{J_o} \frac{\partial J_1}{\partial z} + \frac{J_1 dV_o}{J_o dz} \qquad (8A)$$

whence follows together with (5A),

$$\frac{\partial^2 J_1}{\partial z^2} + \frac{\partial J_1}{\partial z} \left(2 \frac{j\omega}{V_o} + \frac{3}{V_o} \frac{dV_o}{dz} \right) + J_1 \left(2 \frac{j\omega}{V_o^2} \frac{dV_o}{dz} - \frac{\omega^2}{V_o^2} \right) =$$

$$= j\omega \frac{J_o}{V_o^2} \left[\frac{1}{\varrho} \frac{\partial p}{\partial z} \right]_1 \qquad (9A)$$

From a development of the right side of this equation follows

$$\left[\frac{1}{\varrho} \frac{\partial p}{\partial z} \right]_1 = \frac{V_o}{J_o} \frac{\partial p_1}{\partial z} + \frac{1}{j\omega J_o^2} V_o^2 \frac{\partial J_1}{\partial z} \frac{dp_o}{dz} \qquad (10A)$$

Here

$$\frac{\partial p_1}{\partial z} = - \frac{c_o^2}{j\omega} \frac{\partial^2 J_1}{\partial z^2} \qquad (11A)$$

according to equation (3A) and (6A) and according to the equation of force for stationary terms

$$\frac{dp_o}{dz} = - \varrho_o V_o \frac{dV_o}{dz} \qquad (12A)$$

(10A) is now written

$$\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_1 = -\frac{V_o}{J_o}\frac{c^2}{j\omega}\frac{\partial^2 J_1}{\partial z^2} - \frac{1}{j\omega}\frac{V_o^2}{J_o^2}\rho_o V_o \frac{dV_o}{dz}\frac{\partial J_1}{\partial z} \qquad (13A)$$

Substituting this into equ.(9A) and introducing the Mach number $M(z) = V_o(z)/c$ and the wave number $k = \omega/c$ results in

$$\boxed{\frac{\partial^2 J_1}{\partial z^2} - \frac{1}{1-M^2}\left(jkM + M\frac{dM}{dz}\right)\cdot\frac{\partial J_1}{\partial z} + \frac{1}{1-M^2}\left(k^2 - 2jk\frac{dM}{dz}\right)\cdot J_1 = 0} \qquad (14A)$$

If we assume $M \ll 1$, we obtain the differential equation given in the text:

$$\frac{\partial^2 J_1}{\partial z^2} - 2\left(jkM + M\frac{dM}{dz}\right)\cdot\frac{\partial J_1}{\partial z} + \left(k^2 - 2jk\frac{dM}{dz}\right)\cdot J_1 = 0 \qquad (15A).$$

The alternating pressure p_1 is related to the alternating flow density J_1 by

$$p_1 = -\frac{c^2}{j\omega}\cdot\frac{\partial J_1}{\partial z}.$$

Appendix II.

Partial Waves in Resting Air.

The inhomogeneous duct be extending in z-direction. Also the sound wave shall propagate in this direction. F be a sound field value responding to the wave equation

$$\frac{\partial^2 F}{\partial z^2} + (\Delta_{tr} + k^2)F = 0 \qquad (1B)$$

with $k = \omega/c$ (c velocity of sound) and Δ_{tr} the Laplacian operator of the plane normal to the z-axis.

In the homogeneous duct the set-up is made [3]

$$\Delta_{tr}F = -\varepsilon^2 F.$$

with constant ε. When introducing the z-dependent boundary conditions into the general solution of the separated wave equation we come to the condition that ε be z-dependent. Instead of (2B) with constant ε we set up

$$\Delta_{tr} F = - H(z) \cdot F, \tag{3B}$$

where H(z) is a periodic function of z with the length of period L. Whence follows the differential equation

$$\frac{\partial^2 F}{\partial z^2} + (k^2 - H(z))F = 0 . \tag{4B}$$

This is a Hill's differential equation with the periodic coefficient H(z), which has according to the theorem of Floquet a solution of the form

$$F = \sum_n A_n(x,y) \cdot e^{-j\beta_n z} = e^{-j\beta_o z} \cdot \sum_n A_n(x,y) \cdot e^{-j\frac{2\pi n}{L} z} . \tag{5B}$$

The summands represent the partial waves; $A_n(x,y)$ are their amplitudes (complex and depending on the transverse coordinates) β_n with

$$\beta_n = \beta_o + \frac{2\pi n}{L} . \tag{6B}$$

are their phase constants (for dissipation-free propagation; otherwise β_n are the propagation constants), β_o is the phase constant of the fundamental wave. The summation index n runs from $-\infty$ to $+\infty$.

Only the totality of the partial waves is able to satisfy the boundary conditions. On the other hand the mutual ratios of the partial wave amplitudes and of the fundamental wave amplitude are only given by the geometry and the boundary conditions of the duct. This is proved by introducing the solution (5B) into the differential equation (4B), whence is obtained

$$\Sigma\,[\Delta_{tr}A_n + (k^2-\beta_n^2)A_n]\cdot e^{-j\frac{2\pi n}{L}z} = 0. \qquad (7B)$$

These sums form an orthogonal system in $0 \leq z \leq L$, because of

$$\int_0^L e^{-j\frac{2\pi(n-m)}{L}z}\,dz = \begin{cases} L \text{ for } n = m \\ 0 \text{ for } n \neq m \end{cases} \qquad (8B)$$

The summands must therefore disappear seperately. That gives us the following differential equation for the amplitude distribution of the partial waves over the cross-section of the duct

$$\Delta_{tr}A_n(x,y) + (k^2 - \beta_n^2)A_n(x,y) = 0 . \qquad (9B)$$

The mutual amplitude ratios of the partial waves are constant over the entire length of the duct, if the boundary conditions are constant.

From (6B) follows for the phase velocity of the n-th partial wave

$$u_n = \omega/\beta_n = \frac{u_o}{1 + nu_o/fL} = \frac{u_o}{1+n\lambda_o/L} \quad , \qquad (10B)$$

with u_o the phase velocity and λ_o the wavelength of the fundamental wave in the duct. For the group velocity v_{gn} we obtain

$$v_{gn} = \frac{1}{d\beta_n/d\omega} = \frac{1}{d\beta_o/d\omega} = v_{go} \ . \qquad (11B)$$

The energy is transported by the totality of the partial waves.

Appendix III

Partial Waves in Air Flow.

Our problem is: is there in inhomogeneous ducts a partial wave with phase velocity u_n as a solution of the wave equation in flow with the flow velocity V so that

$$u_n = V \tag{1C}$$

The wave equation in friction-free, uni-dimensional flow without vortices with the adiabatic equation of state of air is

$$\left\{\Delta - (1/c^2)[\partial/\partial t + V_o \cdot \partial/\partial z]^2\right\} F = 0 \tag{2C}$$

if the flow velocity is large compared to the alternating velocities [11]. This equation can be transformed into the usual equation by a Galilean transformation

$$\begin{aligned} x &= \hat{x} \\ y &= \hat{y} \\ z(\hat{z},\hat{t}) &= \hat{z} + V\cdot\hat{t} \\ t &= \hat{t} \end{aligned} \tag{3C}$$

This transformation has the disadvantage that, because of

$$\partial/\partial\hat{t} = \partial/\partial t + V\cdot\partial/\partial z \tag{4C}$$

a harmonic analysis is impossible in the new coordinates and that, because of

$$d\hat{z} = dz - V \cdot dt \tag{5C}$$

the length of period $\hat{L}$ becomes time dependent. Therefore subsequently a Lorentz transformation is applied to the transformed coordinates $\hat{z}$ and $\hat{t}$. The wave equation is invariant to the Lorentz transformation and altogether we have

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= \mu\cdot z \\ t' &= (1/\mu)\cdot t + (V/c^2)\cdot z \end{aligned} \tag{6C}$$

$$\mu = 1/\sqrt{1 - M^2} \quad ; \qquad M = V/c \quad .$$

It is

$$\partial/\partial t' = \mu\cdot\partial/\partial t$$
$$\partial/\partial z' = (1/\mu)\cdot\partial/\partial z - (V/c^2)\cdot\partial/\partial t \tag{7C}$$

and

$$dz' = \mu dz$$
$$dt' = (1/\mu)\cdot dt + (V/c^2)\cdot dz \tag{8C}$$

and the wave equation writes

$$\frac{\partial^2 F'}{\partial z'^2} + \Delta_{tr}F' - (1/c^2)\frac{\partial^2 F'}{\partial t'^2} = 0 \tag{9C}$$

Here $F' = F'(x,y,z',t')$ is the field value in the transformed coordinates. Δ_{tr} remains unchanged because of the invariance of x and y. If F is a harmonic function of the time, than also F' is according to equ.(7C) a harmonic function of the time. With the radian frequency ω' and the wave number $k' = \omega'/c$ equation (9C) is written in the transformed coordinate system

$$\frac{\partial^2 F'}{\partial z'^2} + (\Delta_{tr} + k'^2)F' = 0\ . \tag{10C}$$

This is the same equation as (1B) in Appendix II. We can here repeat the set-up (2B):

$$\Delta_{tr}F' = -H'(z')\cdot F' \quad , \tag{11C}$$

since according to (8C) the boundary values are only functions of z'. Therefore H' is a periodic function of z' with the length of period L' and according to equ.(8C) $L' = \mu L$. This means that we can apply the considerations about the equations (4B) to (9B) of Appendix II directly on the corresponding primed values in the transformed coordinate system.

Returning to our initial problem we have to examine whether $u_n' = V'$ is possible. For this purpose we have to express ω', β_o' and L' in

$$u_n' = \frac{\omega'}{\beta_o' + \frac{2\pi n}{L'}} \tag{12C}$$

and also V' in the transformed coordinate system by the corresponding values ω, β_c, L, and V, which are open for measurement.

Applied on $F'(z', t')$ we obtain

$$\partial/\partial z' = -j\beta' \qquad \text{and} \qquad \partial/\partial t' = j\omega'$$

and applied on $F(z, t)$ we have

$$\partial/\partial z = -j\beta \qquad \text{and} \qquad \partial/\partial t = j\omega$$

According to the rules of calculation for the introduction of new coordinates follows

$$\beta' = \beta/\mu + \omega V/c^2 \quad \text{and} \quad \omega' = \mu\omega \ . \tag{13C}$$

With (13C) and $L' = \mu L$ equation (12C) writes

$$u_n' = \frac{\mu^2 u_o}{1 + \mu\frac{u_o V}{c^2} + \frac{nu_o}{fL}} \tag{14C}$$

and from (8C) follows

$$V' = dz'/dt' = V \frac{\mu^2}{1 + \mu M^2} \tag{15C}$$

so that the condition $u_n' = V'$ is equivalent to

$$V = \frac{u_o}{1 + n\frac{\lambda_o}{L}} \ . \tag{16C}$$

This is again the well known condition for synchronism between flow and partial wave. For u_o and λ_c the values with superimposed flow have to be taken.

According to (16C) the partial wave, which with superimposed flow is defined only in the transformed coordinate system by equation (5B) in Appendix II, is propagating with the velocity of flow, if the same was true for a partial wave according to the calculation with resting air as far as the values changed by the flow are introduced for u_o and λ_o. By proper dimensioning equation (16C) can certainly be satisfied.

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